The Influence of External Forces on a Monopolistic Market: a Statistical Physics Perspective

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Abstract

In this thesis, we consider a discrete choice model with a single homogeneous product and a single seller (the monopoly case) in which a population of individuals with idiosyncratic willingness to pay must choose repeatedly to buy or not a unit of this single homogeneous good at a price determined by the monopolist.

Utilities of buyers have positive externalities due to social interactions among customers. If the latter are strong enough, the system has multiple Nash equilibria revealing coordination problems.

We assume that individuals learn to make their decisions repeatedly, and study the performances, along the learning path as well as at the reached equilibria, for different learning schemes based on past earned and/or forgone payoffs. We also calculate the monopolist's profit cumulated during the customers learning process.

We discuss analogies between simulated market mechanisms and classical phenomena in the physics of disordered systems such as phase transition, avalanches, mean-field approximation, quenched and annealed disorder.

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Chapter 1

Introduction

Sellers have always faced the problem of setting the right prices for goods and services that would generate the maximum revenue for them. Determining the right prices to charge a customer for a product or a service is a complex task. It requires that a company knows not only its own operating costs and availability of supply, but also how much the customer values the product and what the future demand would be.

In the simplest market, the buyer has discrete choice (to buy or to not buy) model with a single homogeneous product and a single seller (monopolist).

On the demand side, the customers are assumed to be influenced only by social pressure. On the supply side, the monopolist is a cognitive agent able to set the price to optimize his profits.

Recently [19], analogies between simulated market mechanisms and phenomena occurring in classical statistical physics have been used to study the basic model developed above.

In particular, two different scenarios have been investigated:

(i) individual agent's willingness to pay is assumed to be heterogeneous but fixed.

(ii) the agent's willingness to pay is composed of 2 parts: an homogeneous part (the same for all agents) and an individual additive stochastic part.

These models fall into different classes as far as a physicist is concerned. Whereas the first case corresponds to that of quenched disorder, the second one can be viewed as a problem involving annealed disorder.

These different cases lead to very different consequences for the market behaviour.

After introducing the model (Chapter 2) and recalling some results for the deterministic case (chapter 3 & 4), we are going to develop the improvements we have made to the software MODULECO and the investigate in more details this theory on an empirical study: the Marseille Wholesale Fishmarket.

Chapter 2

The Simplest Model

2.1 Simple Market Model with a Single Good and Externalities

We consider a single good at a price P fixed by a monopolist and a population of N agents (i = 1, 2, . . .,N), with the following characteristics:

Strategies: each individual i has to make a binary choice, that we denote $\omega_i = 1$ (to buy one unit, to adopt a fashion, etc., depending on the situations addressed by the model) or $\omega_i = 0$ (not to buy, not to adopt, etc.).
Each individual has a reservation price, i.e. the maximum price he is ready to pay for the good.

Each customer's willingness to pay is the sum of two terms:

Idiosyncrasy: each individual has his own (idiosyncratic) willingness to pay (called hereafter IWP), H_i; the larger H_i, the higher the willingness to choose the state $\omega_i = 1$. We assume that H_i is distributed among the agents according to a probability distribution function (pdf) of average H and variance θ_i. A uniform distribution was considered by Gordon, but in this paper we consider a logistic pdf.

$$H_i = H + \theta_i \tag{2.1}$$

with θ_i the deviation with respect to the mean, of pdf $f(\theta_i)$.

A **Probability Density Function** (pdf) is a function that represents a probability distribution in terms of integrals. Formally, a probability distribution has density f, if f is a non-negative Lebesgue-integrable function $\mathbb{R} \rightarrow \mathbb{R}$ such as the probability of the interval $[a,b]$ is given by $\int_a^b f(x)dx$ for any two numbers a and b. This implies that the total integral of f must be 1. Intuitively, if a probability distribution has density $f(x)$, then the infinitesimal interval $[x, x+dx]$ has probability $f(x)dx$.

Externalities: the willingness to pay of each individual i increases beyond his IWP (H_i) if a subset ν_i of other agents, called hereafter his neighbours, decide to buy. Given the choices ω_k, $k \in 1, ..., N$, the actual *surplus* of i is:

$$V_i(\omega_k) = H_i + \frac{1}{N_i} \sum_{k \in \nu_i} J_{ik}\omega_k - P, \tag{2.2}$$

where P is the posted price and N_i is the number of "neighbours" of i ($N_i = \|\nu_i\|$). If $V_i(\omega_k)$ is positive (negative), the optimal choice is $\omega_i = 1$ ($\omega_i = 0$). The corresponding utilities are $U_i^{\omega_i} = \omega_i V_i(\omega_k)$. In this chapter, the neighbourhoods ν_i are assumed to be global and homogeneous, that is, every agent is a neighbour of every other agent (complete connectivity: $N_i = N - 1$ for all i) and all the weights of the social component are equal and positive ($J_{ik} = J > 0$ for all $i \neq k$).

Learning: we assume that the individuals do not know the values of the surplus expected upon choosing a strategy ω, but estimate their *attraction* of playing it, A_i^ω, based on their past experiences. The *actual utility* corresponding to a decision ω at time t is:

$$U_i^\omega(t) = \omega V_i(t), \tag{2.3}$$

where the surplus

$$V_i(t) = H_i + J\eta_i(t) - P, \tag{2.4}$$

depends on the actual fraction of neighbours that buy at time t:

$$\eta_i(t) = \frac{1}{N-1} \sum_{k(k \neq i)} \omega_k(t), \tag{2.5}$$

Notice that buying, because the corresponding attraction is large, runs the risk of having a negative utility. But, if the choice is $\omega_i = 0$, the individual has a very small utility but may miss a positive one. There are many possible ways of determining the attractions. In this chapter, we make the assumption that the individuals do not know precisely the values of the parameters H_i and J, and that they may even not know the structure of their utilities. Their estimations rely on the values of $U_i^\omega(t)$ grasped at each period after making decisions.

Simplifying hypothesis

- $J_{ik} > 0 \Leftrightarrow$ making the same choice as the others is advantageous

- The social influence is supposed to be homogeneous ($J_{ik} = J$)

$$\frac{1}{\|v_i\|} \sum_{k \in v_i} J_{ik} \omega_k(t) = J\, \eta_i \tag{2.6}$$

  with J = weight of neighbour's choices and η_i the fraction of i's neighbours that adopt.

- Global neighbourhood and large N:

$$\eta_i = \frac{1}{N-1} \sum_{k=1 k \neq i}^{N} \omega_k \cong \frac{1}{N} \sum_{k=1}^{N} \omega_k \equiv \eta \tag{2.7}$$

  so, in this configuration, η, the fraction of buyers, is insensitive to fluctuations: single agents cannot influence individually the collective term $J\eta$.

2.2 Demand and Supply Sides

2.2.1 The Demand Side

A rational agent chooses ω_i in order to maximize his *surplus function* V_i:

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (H_i + \sum_{k \in v_i} J_{ik} \omega_k - P), \tag{2.8}$$

where P is the price of one unit and H_i represents the idiosyncratic preference component. To simplify we consider the case of homogeneous influences, that is, identical positive weights $J_{ik} = J/n > 0$.

2.2.1.1 Psychological *versus* economic point of view

Depending on the nature of the idiosyncratic term H_i, the discrete choice model (2.8) may represent two different situations.
We can separate a "psychological" and an "economic" approach to individual choice.

Within the psychological point of view of Thurstone [25], the utility has a *stochastic* aspect, referred to as the TP-case (acronym for Thurstone-Psychological). The IWPs present independent temporal fluctuations around a fixed (homogeneous) value.

On the contrary, within the *economic* perspective of McFadden [14], each agent has a willingness to pay that *doesn't vary* in time, at least during the period of consideration, but may differ from one agent to an other. Even if the seller knows the statistical distribution of the IWP over the population, he cannot observe each specific individual IWP. In the language of interactive decision theory, the seller is in a "risky" situation, this is the McF-case (acronym for McFaden).

"Risky" in decision theory and statistics means that because the seller cannot know each specific individual IWP, he has to assess risks and benefits when selling the product. There is a distinction to make between a situation of risk and one of uncertainty. There is an uncontrollable random event inherent in both these situations, but the distinction is that in a risky situation, the uncontrollable random event comes from a known probability distribution, whereas in an uncertain situation the probability distribution is unknown.
The domain of decision analysis models falls between two extreme cases. This depends upon the degree of knowledge we have about the outcome of our actions, as shown below:

Ignorance	Risky Situation	Complete Knowledge
Pure Uncertainty Model	Probabilistic Model	Deterministic Model

The two perspectives (TP and McF) only differ in the nature of the individual willingness to pay, but correspond to very different theoretical models.

In the **TP model** [25], the idiosyncratic preference has two components: a constant deterministic term H (the same for all agents), and a time- and agent-dependent additive term $\theta_i(t)$. The $\theta_i(t)$ are random variables of zero mean and, during the simulations, they are refreshed at each time step. Agent i decides to buy according to the conditional probability:

$$P(\omega_i = 1|z_i(P, H)) = P(\theta_i > z_i(P, H)) = 1 - F(z_i(P, H)), \tag{2.9}$$

with

$$z_i(P, H) = P - H - J_v \sum_{k \in v_i} \omega_k, \tag{2.10}$$

where $F(z_i) = P(\theta_i < z_i)$ is the cumulative distribution of the random variables θ_i.
In this model, the agents make repeated choices, and the time varying components are drawn at each time t from a logistic distribution with zero mean and variance $\sigma^2 = \pi^2/(3\beta^2)$ (the use of this formula for the variance and of a logistic distribution is going to be explained later in this thesis).

$$F(z) = \frac{1}{1 + exp(-\beta z)}, \tag{2.11}$$

In the **McF model** [14], the private idiosyncratic terms H_i are randomly distributed over the agents, but remain fixed during the period under consideration. There are no temporal variations (that means $\forall i,\ \theta_i = 0$). It is useful to introduce, like in the TP model, the notation $H_i = H + \zeta_i$, and to assume that the ζ_i are distributed with zero mean and variance $\sigma^2 = \pi^2/(3\beta^2)$ over the population. It implies:

$$\lim_{N \to \infty} \frac{1}{N} \sum_i \zeta_i = 0 \ \ and \ \ \lim_{N \to \infty} \frac{1}{N} \sum_i H_i = H, \tag{2.12}$$

An agent buys if:

$$\zeta_i > P - H - \frac{J}{N-1} \sum_{k \in v_i} \omega_k, \tag{2.13}$$

In the full connectivity case (model with global externality), it is convenient to identify a *marginal customer*, indifferent between buying and not buying. Let $H_m = H + \zeta_m$ be his idiosyncratic willingness to pay (IWP). He has zero surplus ($V_m = 0$), so:

$$\zeta_m = P - H - \frac{J}{N-1} \sum_{k \in v_i} \omega_k, \tag{2.14}$$

so in this case, an agent buys if $\zeta_i > \zeta_m$, which corresponds to $V_m > 0$ and does not buy otherwise.

There is a strong relation between these models and Ising type models in Statistical Mechanics (cf. 3.2.1), which is made explicit if we change the variables $\omega_i \in \{0,1\}$ into *spin* variables $s_i \in \{\pm 1\}$ through:

$$\omega_i = \frac{1+s_i}{2}, \tag{2.15}$$

In physics, the **TP model** [25] corresponds to a case of **annealed, thermal disorder**. In the particular case where $F(z)$ in (2.9), the distribution of the temporal fluctuations, is the logistic distribution, we obtain an Ising model in a uniform external field $H - P$, at temperature $T = 1/\beta$. In the **McF model** [14], the IWP are randomly chosen and remain fixed, or they present independent temporal fluctuations around a fixed (homogeneous) value: it is analogous to a *Random Field Ising Model* (RFIM) at zero temperature. The RFIM belongs to the class of **quenched disorder** models: the values H_i are equivalent to random time-independent local fields.

Thus, from the physicist's point of view, the TP and the McF models are quite different: uniform field and finite temperature in the former, random field and zero temperature in the latter. Studies show that annealed and quenched disorder can lead to very different behaviours.

The **TP model** [25] is well understood, even if an analytical solution of the optimization problem for an arbitrary neighbourhood does not exist, the *Mean Field* analysis (cf. 2.4) gives approximate results that become exact in the limiting situation where every agent is a neighbour of the $N-1$ other agents (full connectivity).

However, the properties of the **McF model** [14] are not yet fully understood, but several variants of the RFIM have already been used in the context of socio-economic modeling.

2.2.1.2 Demand for a given price

With a "global" externality case, homogeneous interactions and full connectivity, which is equivalent to the *Mean Field Approximation* in physics (cf. 2.4), we consider the penetration rate η, defined as the fraction of agents that choose to buy at the given price. In the large N limit, we have $\sum_{k\in v} \frac{\omega_k}{N-1} \approx \eta$, so that $\theta_m \approx P - H - J\,\eta \equiv z$. This approximation allows us to define η as a fixed point:

$$\eta = 1 - F(z), \tag{2.16}$$

where z depends on P, H, and η.
Using the logistic distribution for θ_i, we have:

$$\eta = \frac{1}{1 + exp(+\beta z)}, \tag{2.17}$$

This equation is formally equivalent to the individual expectation that $\omega_i = 1$ in the TP case.

Thanks to the equation (2.16), we can define the penetration rate as an implicit function of the price:

$$\Phi(\eta, P) \equiv \eta(P) + F(P - H - J\,\eta(P)) - 1 = 0 \tag{2.18}$$

For a given P, equation (2.16) defines the penetration rate η as a fixed point, so the inversion of this equation gives us an *inverse demand function*:

$$P_d(\eta) = H + J\eta + G(\eta) \tag{2.19}$$

where $G(\eta)$ is the inverse of the complementary distribution function; it satisfies:

$$\int_{G(\eta)}^{\infty} f(x)\,dx = \eta, \tag{2.20}$$

2.2.2 The Supply Side

The mutual interactions between customers introduce multiple solutions in the demand function and are responsible for the existence of a transition in the optimal strategy of the monopolist.
On the supply side, we assume that the monopolist does not know the idiosyncratic willingness to pay of each customer, but he is aware of its distribution among the population. He has to determine the best price to optimize his profit. Since the demand may be a multiple valued function of the price, the monopolist's situation is risky.
However, he cannot observe any *individual* reservation price. He only observes the aggregate result of the individual choices (to buy or not to buy), the fraction of customers η.

2.2.2.1 Profit Maximisation

The system exhibits a (sudden) first order phase transition with respect to the price in the profit optimisation by the monopolist: if the social influence is strong enough, there is a regime where, if the mean willingness to pay increases, or if the production

costs decrease, the optimal solution for the monopolist jumps from a solution with a high price and a small number of buyers, to a solution with a low price and a large number of buyers.
Depending on the path of prices adjustments by the monopolist, simulations show hysteresis effects on the fraction of buyers (cf. 3.3.2).

Let $c \equiv C/\sigma_H$ be the monopolist's cost in units of σ_H (the variance of the distribution of the IWP) for each unit sold and p the monopolist's price of one unit, and let $p - c$ be his normalized profit *per unit*.

Since each customer buys a single unit of the good, the monopolist's total expected profit is $(p - c) * N * \eta$. Thus he has to solve the following maximisation problem:

$$p_{Max} = arg \max_p \Pi(p), \tag{2.21}$$

where $N\,\Pi(p)$ is the expected profit, with:

$$\Pi(p) \equiv (p - c)\,\eta(p), \tag{2.22}$$

and $\eta(p)$ is the penetration rate defined as the fraction of customers that choose to buy at a given price. If there is no discontinuity in the demand curve $\eta(p)$ (i.e. for $J < j_B$), p_{Max} satisfies $d\Pi(p)/dp = 0$, which gives $d\eta/dp = -\eta/p$ at $p = p_{Max}$, and using the equation (2.16) to calculate the derivative, we obtain at $p = p_{Max}$:

$$\frac{f(z)}{1 - Jf(z)} = \frac{\eta}{p}, \tag{2.23}$$

where z has to be taken at $p = p_{Max}$.

Because the monopolist observes the demand level η, we can use equation (2.16) to replace $1 - F(z)$ by η. We obtain the monopolist's price as a function of the demand, the effective inverse supply function:

$$p_s(\eta) = c - \eta[G'(\eta) + J], \tag{2.24}$$

We obtain p_{Max} and η_{Max}, the corresponding fraction of buyers, as the intersection between supply (2.24) and demand (2.19):

$$p_{Max} = p_s(\eta_{Max}) = p_d(\eta_{Max}). \tag{2.25}$$

The monopolist's supply price is the solution of this equation which maximizes his profit.

If $f(x)$ is differentiable, the maximum satisfies:

$$\frac{d^2\Pi}{dp^2} < 0. \tag{2.26}$$

In the case of multiple extrema, the one which maximises Π has to be selected.
For $J > j_B$, the monopolist has to find $p = p_{Max}$ which realises the program:

$$p_{Max} : \max\left(\Pi_-(p^-_{Max}), \Pi_+(p^+_{Max})\right) \tag{2.27}$$

$$p^+_{Max} = arg\ \max_p \Pi_+(p) \equiv p\,\eta_+(p) \tag{2.28}$$

$$p^-_{Max} = arg\ \max_p \Pi_-(p) \equiv p\,\eta_-(p) \tag{2.29}$$

where the subscript + (-) refers to the solution of (2.16) with the largest (smallest) fraction of buyers. There are two ways to maximize the profit for the monopolist: to sell to a small amount of customers at a high price or to sell to a large amount of customers but at a reduced price. The reason for that is because of the stability (equilibrium) of these two states in the evolution of the market. If the price is between these two states, it will tend to change quickly to one of these two equilibria. The stability of the equilibria opposed to the temporary character of the other possible solution is the reason why we can assume that there are only two solutions to the previous program.

2.3 Learning By the Customers

The fundamental question in the agent-based theory is: which model describe human behaviour best? In order to understand the process of learning by agent in many multi-agent platforms, and especially Moduleco which was used to run the simulations, we are going to investigate some of them and have a look at their differences. This process was necessary to verify that the method used in the software Moduleco for the agents to learn was appropriate to the assumptions made. Even if there are many papers relating to this subject, we are only considering one general model that can be split into several models by taking different values of the parameters. Actually, we can consider different learning rules in Moduleco, based on this particular algorithm, and the aim of this section was to have a better understanding of this process, and so avoiding to use a random learning rule which can be not applicable to our model.

Like Camerer [3] and his Experience Weighted Attractions (EWA) scheme, we consider a general family of learning rules, which allows us to represent in a single expression many learning rules proposed in the literature and studied in the following pages ([5], [21],

[22]). The EWA is a general model. It combines elements of two different approaches. One approach starts with the premise that agents keep track of the history of previous behaviour of the other agents and form some belief about what others will do in the future based on past observation. Then they tend to choose a strategy that maximizes their expected payoffs given the beliefs they formed. A different approach assumes that strategies are reinforced by their previous payoffs, and the propensity to choose a strategy depends in some way on its own stock of reinforcement. Agents who learn by reinforcement do not generally have belief about what other agents will do.
Given the actual utility $U_i^\omega(t)$ of strategy ω, in Moduleco, each agent uses the following adaptive rule to update the attractions:

$$A_i^\omega(t+\tau) = [1-\mu(t)]A_i^\omega(t) + \mu(t)\Delta_i(t)U_i^\omega(t), \tag{2.30}$$

$$\Delta_i(t) = \delta + (1-\delta)I[\omega_i(t), \omega], \tag{2.31}$$

$$\mu(t+\tau) = (1-\kappa)\frac{\mu(t)}{\mu(t)+\phi} + \kappa(1-\phi) \tag{2.32}$$

where $I(x,y)$ is the indicator function ($I(x,x) = 1,\ I(x,y) = 0$ for $y \neq x$) and τ is the elementary time step.
The factor $\Delta_i(t)$ allows to update differently the attractions of played and non-played strategies. These strategies will not have the same impact on the behaviour of the agent for the next choice he will have to make. The influence of *delta* in its formula is fundamental because it measures the relative weight given to foregone payoffs compared to actual payoffs, in updating attraction.
$\mu(t)$ is usually called *learning rate, discount factor or depreciation rate of past experience* in statistical learning theory. It weights the relative importance of recent payoffs with respect to the past estimations (the most recent information are more important than the others). The values of the learning parameters $\mu(0) > 0$, $\kappa \in \{0, 1\}$, $\phi \geq 0$ and $0 \leq \delta \leq 1$ in (2.30), (2.31) and (2.32) correspond to different assumptions about the rationality and cognitive capacities of the customers. We can parametrise the model by choosing different values of the parameters when we create our models: $\mu(0)$, μ, κ, ϕ, δ. These parameters allow to decline different learning algorithms.

The abcve equations may be simplified within the binary-choices framework of our model. First, since $\omega \in \{0, 1\}$, we may write $\Delta_i(t) = \delta + (1-\delta)\omega_i(t)$ in (2.31). Moreover, since the utility of strategy $\omega = 0$ is strictly zero, the corresponding attraction converges to 0 asymptotically, independently of the past decisions. Any rational customer initializes $A_i^0(0) = 0$, without the need of learning. We only consider in the following learning the attractions for buying, A_i^1. In the case of binary choices, after the strategy $\omega_i(t)$ is played, the attraction for buying in next period is estimated as follow:

$$A_i^1(t+\tau) = [1-\mu(t)]A_i^1(t) + \mu(t)[\delta + (1-\delta)\omega_i(t)]U_i^1(t), \tag{2.33}$$

$$\mu(t+\tau) = (1-\kappa)\frac{\mu(t)}{\mu(t)+\phi} + \kappa(1-\phi) \tag{2.34}$$

where $U_i^1(t)$ is the actual utility of strategy $\omega = 1$ at period t.

2.3.1 Decision Rules

Decisions are taken based on the learned attractions. We assume that each agent chooses a strategy ω at time t using a probability law that depends on his attraction for buying at time t, $P(\omega(t)|A_i^1(t))$. Several decision rules have been used in the literature, but here we explicit them in the case of binary decisions.

2.3.1.1 Myopic Best Response

Myopic Best Response selects the strategy optimizing the expected utility:

$$\omega(t+\tau) = \Theta(A_i^1(t)) \tag{2.35}$$

where Θ is the heavyside function (equal to 1 if $x > 0$, 0 otherwise). The decision depends then only on the sign of the attraction and not on its magnitude: the individual buys whatever the value of the attraction provided is positive. This response is optimal *with respect to the attractions.*

In the special case where the attractions are equal to the utility earned at the preceding period this decision rule coincides with what is usually called *Cournot Best Reply* in the literature.

2.3.1.2 Trembling Hand

The adopted strategy is selected using the following probabilistic decision rule:

$$P_\beta(\omega(t+\tau)) = 1) = 1 - \epsilon(t) \tag{2.36}$$

$$P_\beta(\omega(t+\tau)) = 0) = \epsilon(t) \tag{2.37}$$

where ϵ is a noise parameter ($0 \leq \epsilon \leq 1$). In our case of binary decisions, we have:

$$\epsilon(t) = \frac{1}{1 + exp\beta A_i^1(t)} \tag{2.38}$$

This rule reduces to the myopic best response if $\beta \to \infty$.

2.3.2 Learning Rules

We can generate different learning rules. Although we consider the special case of binary decisions, where the agents only need to estimate the attractions for buying, the discussion that follows is very general, easily transposable to situations with more strategies.

$\mu(t)$ sets the memory decay rate of past attractions. This decay may arise because of limited memory capacity or because the agent believes that older information may not be as relevant as the new one. It is parametrized by the values of κ and ϕ, which control the time dependence of the learning rate.

2.3.2.1 Myopic Learning

For $\phi = 0$ and any $\kappa \in \{0, 1\}$, $\mu(t) = 1$ and (2.4) gives:

$$A_i^1(t+\tau) = [\delta + (1-\delta)\omega_i(t)]U_i^1(t), \tag{2.39}$$

This is **myopic learning** since the attraction at each time step t only relies on the outcome of the preceding iteration, without keeping any trace of the previous steps.

The value of δ allows to update the attraction of the played strategy in a different way from that of non-played ones. If $\delta = 1$, all the strategies are equally updated, a learning scheme known as *fictitious play*. If $\delta = 0$, we get *reinforcement learning*: the attraction for buying is only updated after buying, that is to say only if it is positive. Otherwise, it remains negative and its absolute value decays by a factor $1-\mu(t)$ at each period.

2.3.2.2 Time-averaged Learning

If $\kappa = 0$ and $\phi > 0$, the learning rate decreases through time. Moreover, equation (2.5) gives directly:

$$\mu(t+\tau) = \frac{\mu(0)(1-\phi)}{\mu(0)(1-\phi^t) + \phi^t(1-\phi)} \tag{2.40}$$

If $\phi < 1$, $\mu(t)$ converges in the limit of $t \to \infty$ to $1 - \phi$, the same time independent learning rate as when $\kappa = 1$. This convergence is faster the smaller the value of ϕ, and for $\phi = 0$, the value $\mu = 1$ is reached after only one time step: in that particular case, only the last utility determines the attraction, like in myopic learning.

2.3.2.3 Time-decay Learning

When $\phi > 1$, the learning rate decreases asymptotically like ϕ^{-t}, so learning becomes less and less effective with time. A too fast decrease of $\mu(t)$ may stop prematurely the learning process, whereas excessively large values of $\mu(t)$ may induce a chaotic behaviour. Small values of ϕ (but greater than 1) are preferable for successful learning, at the price of long learning times.

2.3.2.4 Weighted Belief Learning

When $0 < \delta < 1$, we have *weighted belief learning*: the utility of the strategy actually played at time t has a greater influence on updating the corresponding attraction than the potential utility of non-played strategies have on their own attractions. As a consequence, non-buyers will systematically underestimate the absolute value of the attraction for buying.

2.4 Mean Field Approach

The type of connectivity has a strong influence on the model's evolution, that is the reason why to suppose the extreme case with full connectivity is making the model less realistic.

Neighbourhood	(a) No relation	(b) Localised relations	(c) Generalised relations
Level of interactions	independent agents	Localized interactions	Global interactions
Sensitivity to the network topology	Null	Strong	Null
Avalanches	No	Localized in the network	not localized in the network

Table 2.1: A typology of interactions and demand dynamics.

In the first case (a), there is no relation between agents, it corresponds to the case of each agent doesn't have any neighbours. In this case, the aggregate demand doesn't depend on any interaction structure, and there is no external effect (local or global). The agents are independent to one another.

In the second case (c), all agents interact by means of global interactions (e.g. the rate of adoption in the whole population), or full connectivity. Let $\eta \equiv N_a/N$ be the rate of adoption within the population. For N sufficiently large, this rate is close to the rate of adoption within the neighbourhood of each agents: $\eta \simeq N_a/(N-1)$.

This case corresponds to the *Mean Field Approximation* in statistical physics.

The intermediate case (b) corresponds to situations where agents have specified relations, but their neighbourhood can be regular or not. Not all agents are directly connected to one another. This local interdependence gives rise to localised avalanches in the network (cf. 3.2.1).

A many-body system with interactions is generally difficult to solve exactly because of the treatment of combinatorics generated by the interaction terms in the Hamiltonian when summing over all states. The aim of mean field theory is to solve these combinatorial problems.

The main idea of this theory is to replace all interactions with an average or effective interaction, to replace randomly fluctuating quantities by their expectation, thus neglecting fluctuations. The effect of interaction is incorporated into the average field produced by all the other spins and small influence on an agent of the system will not be material. In field theory, the Hamiltonian may be expanded in terms of the magnitude of fluctuations around the mean of the field. The Mean Field Theory can so be viewed as the simplest model expansion of the Hamiltonian in fluctuations. Physically, it means that the system has no fluctuations, but it coincides with the idea that one is replacing all interactions with a mean field.

The main idea of the Mean Field Approximation is, when considering a particular agent i, to neglect the fluctuations of the agents interacting with i. The resulting system behaves as one composed of independent variables, and this independence allow us to factorize the probability density function. More specifically, for all j different from i, the J_j are fixed to their mean value. However, these mean values are unknown and it is actually the goal of the approximation to compute them. Therefore, the method depends on a self-consistency condition which is that the mean computed based on the approximation must be equal to the mean used to define this approximation. An approach consists of performing a perturbation theory with the mean field model as the reference model of zeroth order model. In this approach, we assume that the fluctuations are small and so the interactions with individual nodes are neglected and replaced by a mean field.

To illustrate this theory, we can explore a generalization of this approach to our problem by writing a differential equation describing the time evolution of one J_j, this J_j will be for instance the profit obtained the last time the buyer dealt with the j^{th} seller, or it may be some moving average value of past profits from seller j:

$$\frac{dJ_j}{dt} = -\gamma J_j + < \pi_j >, \tag{2.41}$$

where $< \pi >$ is the average profit, related to π the profit obtained from one actual transaction (the price at which the monopolist sells the good less the cost to produce or buy it at first). γ is a scaling coefficient always smaller than 1 in order to take into account that the events far in the past have to be progressively forgotten. $< \pi >$ is obtained as follow:

$$< \pi_j >= \pi P(j) \frac{exp(\beta J_j)}{\sum_k exp(\beta J_k)}, \tag{2.42}$$

where the fraction represents the probability that a buyer i visits a seller j and $P(j)$ is the probability that the shop still has goods to sell when he comes. We suppress here the i index corresponding to the buyer. In other words, the above set of equations couples the evolution of all the J_j. Equilibrium values are obtained by equating the derivatives to zero.

Instead of using a real *oligopolistic* market as this is the case in the Marseille wholesale fishmarket, we are here considering a simpler case of two shops, a *duopoly*, and to further simplify computation, we suppose that $P(j) = 1$, which happens when buyers always find what they require at the seller they visit. It means that the seller always have in stock what the buyer need which is of course not the case in reality and is influencing the price of the good as well as we will see in the Chapter 6. The equilibrium relations are in this particular case:

$$< \gamma J_1 >= \pi \frac{exp(\beta J_1)}{exp(\beta J_1) + exp(\beta J_2)}, \tag{2.43}$$

$$< \gamma J_2 >= \pi \frac{exp(\beta J_2)}{exp(\beta J_1) + exp(\beta J_2)}, \tag{2.44}$$

Subtracting these two equations, we see that the difference between the two fidelities, $\Delta = J_1 - J_2$, obeys the following implicit equation:

$$< \gamma \Delta >= \pi \frac{exp(\beta \Delta) - 1}{exp(\beta \Delta) + 1}, \tag{2.45}$$

Actually, the right side of the equation is the hyperbolic tangent of $\beta\Delta/2$. The above equation has either one or three solutions according to the slope of the hyperbolic tangent at the origin. By developing the hyperbolic tangent in series for small values of $\beta\Delta/2$, it is easily seen that for:

$$\beta < \beta_c = \frac{2\gamma}{\pi}, \tag{2.46}$$

There is only one solution $\Delta = 0$ and $J_1 = J_2 = \frac{\pi}{2\gamma}$. Since in this case the average J_j are small and equal, the probabilities of visiting either shop simply fluctuate. This is due to an other parameter not taken in consideration in this thesis but that is noticeable in the empirical study of the Marseille FishMarket; the physical distance between the different shop. This distance may not be the same from one day to the other and some non-faithful buyers are only randomly making their choice at the first position; No order is observed. In the opposite situation, when β is above β_c, the zero solution is unstable and one obtains two symmetrical solutions where one fidelity is larger than the other one by a factor which is exponential in $\frac{\beta\pi}{\gamma}$. The transition between the two regimes is abrupt. A development in series of the hyperbolic tangent around 0 shows that the larger fidelity increases in β as the square root of the distance to the transition:

$$\Delta = \sqrt{\frac{24(\beta - \beta_c)}{\beta^3}}, \tag{2.47}$$

Fidelities are then continuous across the transition, but they increase (or decrease) with an infinite slope at the transition. Expression (2.46) can be generalized to any number n of shops:

$$\beta_c = \frac{n\gamma}{\pi}, \tag{2.48}$$

The above analysis shows that as long as the mean field approximation remains valid, the qualitative behavior of the dynamics, ordered or disordered, only depends on one parameter: the ratio between β and β_c. All other parameters simply change the scale of profits, prices, numbers of shops and customers. The time scale of learning depends on γ: order, when achieved, is reached faster for larger values of γ.

The three parameters π, β and γ are so controlling the transition. Sellers set prices and thus determine π, the buyers' profit. The buyers characteristics determine β and γ. We might assume that agents are not all identical and that their characteristic parameters vary. Prices may not vary widely since there is competition between sellers, and so if a seller wants to make profits he has to adapt his prices to be competitive and to avoid having to lower his prices later. On the other hand, memory (characterised by γ) and discrimination rate (characterised by β) might differ between buyers. If these variations are large enough, we might expect to observe two distinct classes of buyers: faithful buyers, who most of the time visit the same shop, would be those whose parameters are such that $\beta > \beta_c$, while searchers with parameters such that $\beta < \beta_c$ would wander from shop to shop without finding the best seller (in respect with the price, and if the seller has good with the quality they are looking for). Indeed precisely this sort of "division of labour" is observed on the Marseille fish market which was one of the empirical points for this thesis: some buyers are faithful, i.e. are going to the same seller when they want to purchase a good, and others are trying several times in different sellers to find the best value/product for them, but this research is made randomly because the parameter β is not strong enough, even if we will see at the end of the thesis with the proper study of the Marseille fish market that some unfaithful buyers are sometimes buying the good where there are a lot of buyers already, but their favourite seller can change from one day to another.

2.5 Maximum Entropy Principle

In most real life situations, the probability distributions of random variables $\mathbf{x}$ are unknown. In elementary situations, it is possible to *assume* a distribution, based on considerations of symmetry or other *a priori* knowledge (we may know an average value or some global constraints). In some cases, supplementary information can be obtained through measurement or observations. If we ask the following question:

What is the probability distribution of the possible states $\mathbf{x}$ of a system, $P(x)$ compatible with our measurements and/or our prior knowledge ?

The *maximum entropy principle*, first introduced by Shannon in 1948 and Jaynes [11] in 1957, give the following answer:

Among all the distributions that are compatible with the constraints (usually, empirical facts like conservation of energy in mechanical systems, experimental results, etc...), the probability $P(x)$ that contains *the maximal* available information is the one that maximizes the entropy.

In other words, the less biased distribution that encodes certain given information is that which maximizes the information entropy, it does not include any information besides that carried by the available data or our prior knowledge.

Chapter 3

Theoretical analysis

3.1 Ising Model and Disorder

3.1.1 Ising Model

The Ising model has been proposed in 1925 [10] to explain the physical properties of magnets. These are a consequence of interacting magnetic moments carried by the elementary particles (electrons and protons) that constitute the molecules of the solid. The magnetization of a piece of condensed matter is a macroscopic observable, obtained by adding the contributions of the molecular moments, called *spins* hereafter. If each spin adopted any arbitrary orientation, the sum would be vanishing small. This is indeed the case of most materials around us (glass for instance).

In presence of a magnetic field, magnetic moments exhibit a "preferred" orientation (the one that minimises the moments' magnetic energy). They become aligned parallel to the field, and a macroscopic magnetic moment is observable. One of the challenging questions at the beginning of last century was to explain why some materials, like Iron, present a permanent magnetization in the absence of external magnetic fields. The Ising model allows to understand how such a collective state may appear, due to very strong quantum mechanical interactions between moments.

The model considers N spins s_i ($i = 1, ..., N$) that may be oriented either up ($s_i = 1$) or down ($s_i = -1$) *vectorially*. The binary spins of the Ising model allow for a scalar notation; sums of such vectors are simple algebraic sums.

In the absence of spin interactions, the energy of an individual spin s_i in an external magnetic field is:

$$E_i = -hs_i, \tag{3.1}$$

A spin parallel to h has energy $-h$, while if it is antiparallel, the energy is higher: $+h$. The total energy of a system of N non-interacting spins in an external magnetic field h

is the sum of the individual energies:

$$E = -\sum_{i=1}^{N} h s_i, \tag{3.2}$$

In the state of minimal energy, called *fundamental state*, the spins are all parallel to h: the sign of s_i is that of h. The total magnetization M is:

$$M = \sum_{i=1}^{N} s_i = N_+ - N_-, \tag{3.3}$$

where N_+ is the number of spins parallel to h, and $N_- = N - N_+$ is the number of spins antiparallel to h. Now, consider a system where the spins interact with each other. By interaction we mean that each spin produces an effective microscopic magnetic field called *exchange field*, on the others. The field produced by spin k on spin i is proportional to the spin's own magnetic moment: $J_{ik}s_k$. The constant of proportionality J_{ik}, called *exchange constant* in physics, represents the strength of the interaction. If $J_{ik} > 0$, it is *ferromagnetic*. It favors that spin i be oriented parallel to s_k. If $J_{ik} < 0$, the interaction is *anti-ferromagnetic* and favours antiparallel alignment.

The *local field* acting on spin i is the sum of the fields produced by its neighbours and the external field h,

$$h_i = \sum_{k=1}^{N} J_{ik} s_k + h, \tag{3.4}$$

The spin's energy $E_i = h_i s_i$ depends on the orientation of the neighbours through the exchange field $J_{ik}s_k$. The total energy of a system of interacting Ising spins is:

$$E = -\frac{1}{2} \sum_{i,k=1}^{N} J_{ik} s_i s_k - h \sum_{i=1}^{N} s_k, \tag{3.5}$$

were the factor $\frac{1}{2}$ is introduced to compensate for the double counting of each couple of spins (i, k) in the sum over i and k. Now the spins' orientations in the state of minimal energy, which are parallel to their local fields, cannot be as easily determined as before, due to the interactions.

In the case of non-interacting spins, $J_{ik} = 0$ and we recover previous equations: the energy is the sum of the energies of the independent spins.

3.1.2 Quenched/Annealed Disorder

By analogy with physical processes, we talk about quenched disorder when some parameters defining a system's behavior are random variables which do not evolve with time, i.e. they are quenched or *frozen*. As a typical statistical physics example, we may cite spin glasses.

It is opposite to annealed or moving disorder, where the random variables are allowed to evolve themselves (parameters take part in dynamics with the same type of state variables). The evolution of a system presenting an annealed disorder is related to that of the degrees of freedom defining the system. These systems are usually considered to be easier to deal with mathematically, since the average on the disorder and the thermal average may be treated on the same footing.

The IWP of the agent consists of a component h_i common to all the agents and into an idiosyncratic component θ_i which represents the diversity of taste between the agents. The fixed idiosyncratic component θ_i is supposed to be distributed between the agents according to a symmetrical law of probability of density $f(\theta)$, a repartition function $F(\theta)$ and zero mean.
The agents are positioned randomly on a network with fixed structure, that we can describe as a random field. As this field is fixed, the physicists qualify this situation of quenched disorder. We can oppose it to the other alternative model, where θ_i are random. In this case, we are dealing with Markovian Random Fields and the corresponding disorder is described as annealed by the physicists.

3.2 Avalanches and Hysteresis

In the presence of externality, two different situations may exist, depending on the price: one with a small fraction of adopters and one with a large fraction. The jump in the number of buyers occurs at different price values according to whether the price increases or decreases, leading to *hysteresis loops* as presented below.

3.2.1 Avalanches

The term *avalanche* is associated with a chain reaction where the latter is directly induced by the behavioural modification of one or several other agents and not directly by the variation in cost. The cost influence is only indirect. For example, in the left part of the Table 3.1, an external cost variation (the same for all agents: C to C') induces a simultaneous (but independent of all social influence) change of two agents i and j (connected one to the other or not). Thus, the mechanism is directly related to the cost and is independent to the social network. If, on the other hand, the cost variation induces the behavioural change of agent i, and therefore, because of agent i changes his behaviour, then agent j changes also his behaviour by social effect without any new change in cost, by "domino effect". In that case, the cumulative effect of a chain of such induced influences is called an "avalanche".

Direct effect of price	Indirect effect of price (social influence : avalanche)
variation in cost (C $\longrightarrow$ C') $\swarrow$ $\searrow$ Change of agent i Change of agent j	variation in cost (C $\longrightarrow$ C') $\downarrow$ Change of agent i $\downarrow$ Change of agent j

Table 3.1: Direct and indirect effect of prices upon individual choices.

3.2.2 Hysteresis

Another important qualitative result of the mean field approach is the existence of hysteresis effects: buyers might still have a strong preference for one shop that offered good deals in the past, even though the current deals they offer are less interesting than those now offered by other shops. The adoption by a single "direct adopter" may lead to a significant change in the whole population through a chain reaction of "indirect adopters". The jump in the number of adopters occurs at different cost values according to whether the costs increases or decreases, leading to *hysteresis loops* as presented below. If the *Idiosyncratic Willingness to Pay* (IWP) is the same for all agents ($H_i = H$, for all i), the model would be equivalent to the (quenched) Ising Model with an "uniform external field": $H - C$. In such case, we would have a "first order transition" with all the population abruptly adopting as $H \geq P$. In the figure below, this initial (decreasing) threshold is: $P_{min} = H$, where the whole population abruptly adopts. After adoption, the increasing cost threshold is: $P_{max} = H + J$, where the whole population abruptly chose $w_i = 0$ (for all i). When agents are adopters, cost variations between P_{min} and P_{max} have no effect on the agents choice. Within that zone $[P_{min}, P_{max}]$, there are two possible equilibria for a given cost.

From a theoretical point of view, there is a singular price $P^* = H + J/2$ (the center of the interval $[P_{min}, P_{max}]$, which corresponds to the *unbiased situation*, where the

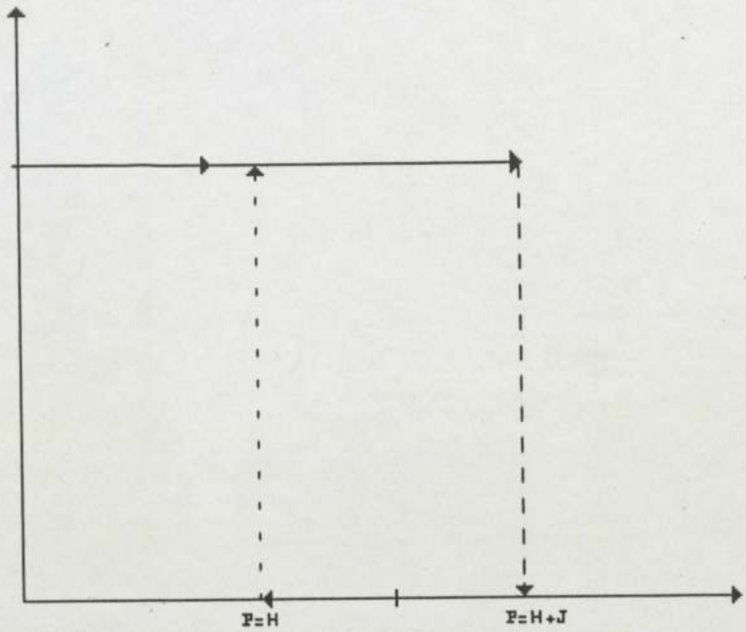


Figure 3.1: Hysteresis with uniform Idiosyncratic Willingness to Pay ($H_i = H$)

willingness to pay is neutral on average, there are as many agents likely to buy or not to buy ($\eta = 1/2$). Let suppose that we start within a similar network in such a neutral state. The agents makes their initial choice on the basis of some prior expectation about the number of adopters and further choice by updating this prior by use of the observed outcome (cf. 2.3.2). Assuming first that all the agents have the same expectation $\eta_i^e = \eta^e$ for all i, each agent has a willingness to pay equal to: $H + J\eta^e - P^* = J(\eta^e - 0.5)$. If $\eta^e > 1/2$, the expected surplus is positive and all agents adopt. Then, the surplus will be $J/2$. Conversely, if $\eta^e < 1/2$, the expected surplus is negative and no agent chooses to buy the product. The final result is similar if we have two classes of people with heterogeneous expectations. Those with $\eta_i^{e+} > 1/2$ (in proportion α) adopt. If $\alpha > 1/2$, the percentage of adopters is such as pessimistic agents also adopt, and so on until complete adoption (and inverse process for $\alpha < 1/2$. This critical point plays a central role in the so called *spontaneous symmetry breaking*, even when agents are only locally connected. As in our simple example, the collective equilibrium state become identical to the individual state: either all agents adopt, or no agent adopts (cf. [6]).

So a given variation in price may induce one or more adoption(s) in the neighbourhood of an agent, and therefore, through social influence, the change in the consumer willingness to pay may lead this agent to adopt indirectly. The most spectacular result in avalanches may be observed when all agents update their choices simultaneously (Figure 3.2).

The Figure 3.2 illustrates the fact that the variation in price may have an influence on the behavior of the agent but the social influence is stronger and that is the reason why we can observe a first order phase transition with respect to the price, a sudden adoption by most of the agent at a precise step. These adoptions are not due to the change in price directly but more to the behaviour of the neighbours of the agent that choose to buy at this step.

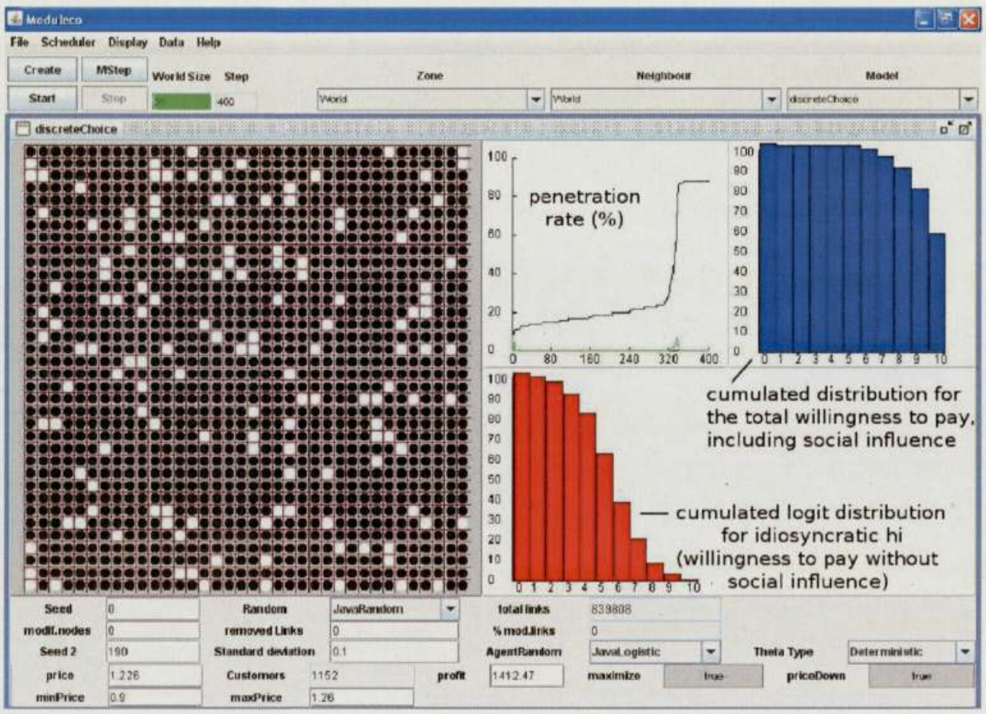


Figure 3.2: First order phase transition under "world" activation regime. (modeled with Moduleco)

3.3 Different Phases

After introducing the different type of phase transitions, we are looking at a model which describes the properties of many different systems (physical as well as social). This type of study has already been carried out for various network architectures. In the presence of externality, and depending on the parameters, two different stable equilibria - or "phases" - may exist for a given cost: one with a small fraction of buyers (in some case with no adopter) and one with a large fraction (in some cases, everybody adopts). By an external variation of the cost, a transition may be observed between these phases.

3.3.1 Phase Transition

In thermodynamics, a phase transition is the transformation of a thermodynamic system from one phase to another. A great diversity of economic and social phenomena, strongly depending on the social interactions, show similar properties as thermodynamic system as for their dynamics and stationary states.
The three following characteristic facts are observed :

- abrupt transitions: the passage from one state to another is sudden (it could be with respect to the price, to the time, to the temperature, etc...).
- stability: once the new state installed, it appears very stable.
- coexistence of equilibrium: in spite of similar economic and social conditions, different equilibria may be observed.

Robustness and coexistence are the essential ingredients responsible for a phenomenon of hysteresis, as it has been seen in the previous section.

In the modern classification scheme, phase transitions are divided into two broad categories:
The **first order phase transitions** are those that involve a latent heat. During such a transition, a system absorbs or releases a fixed among of energy and during this process, the temperature of the system will stay constant as heat is added.
The second class of phase transitions are the **second order phase transitions** also called **continuous phase transitions**. These have not latent heat.

In these systems, there exists a special combination of parameters, known as critical point, at which the transition between two states becomes a second order transition. Near the critical point, the distinction between the two different phases (or states) is almost non-existent.

3.3.2 Multiple Solutions and Customers Phase Diagram

We consider the simplest system where the individuals willing to pay large prices are fewer than those willing to pay low ones. This is a population where H_i, the individuals' IWP distribution is triangular around its mean value H, such that the fraction of individuals with a given H_i decreases linearly with H_i.

With normalized parameters $h \equiv H/\sigma_H$, $x_i = (H_i - H)/\sigma_H$, where σ_H is the variance of the IWP distribution, the random variables x_i have zero mean and unitary variance. As it has already been used in the work from V. Semeshenko and J.P. Nadal [22], we can show here an example with a triangular probability density function given by:

$$f(x) = \begin{cases} 0 & \text{if } x \leq -\sqrt{2} \\ \frac{2(2\sqrt{2}-x)}{18} & \text{if } -\sqrt{2} \leq x \leq 2\sqrt{2} \\ 0 & \text{if } 2\sqrt{2} \leq x \end{cases} \tag{3.6}$$

This triangular distribution is used in this section because it presents the advantage of allowing an analytical determination of the system's equilibrium properties, but this distribution is more theoretical than practical. Some more recent studies ([23]) are using a logit IWP distribution that are more realistic but harder to understand and this type of graph would be only complicating the explanation about the multiple solutions.
The fraction of buyers is given by the solutions of the equation $\eta = \int_{p-h-j\eta}^{2\sqrt{2}} f(x)dx$.
For each posted price the monopolist's profit is $\Pi(\eta) = \eta(p - c)$ where c is the unitary cost, i.e. the cost of one unit of the good. We can so, as usually described in the literature about economics ([1], [24]), visualize the properties of the system on a Phase Diagram, and so still make the parallel with physics.

We can see on the Figure 3.3 the customers phase diagram. V. Semeshenko and J.P. Nadal [22] analysed that there are different equilibrium states for different values of the normalized parameters $j \equiv J/\sigma_H$ and $p - h \equiv (P - H)/\sigma_H$. For $j = 0$, if the price p is larger than the maximal IWP in the population ($p > h + 2\sqrt{2}$), then nobody buys and $\eta = 0$. On the contrary, when it falls below the smallest IWP in the population ($p < h - \sqrt{2}$), all the customers buy the product ($\eta = 1$). For intermediate prices, $\eta(j = 0)$ decreases with the price:

$$\eta(j = 0) = \frac{(h + 2\sqrt{2} - p)^2}{18} \quad for \quad -\sqrt{2} \leq p - h \leq 2\sqrt{2} \tag{3.7}$$

and saturates at $\eta = 0$ for $p - h > 2\sqrt{2}$; $\eta = 1$ for $p - h < -\sqrt{2}$.

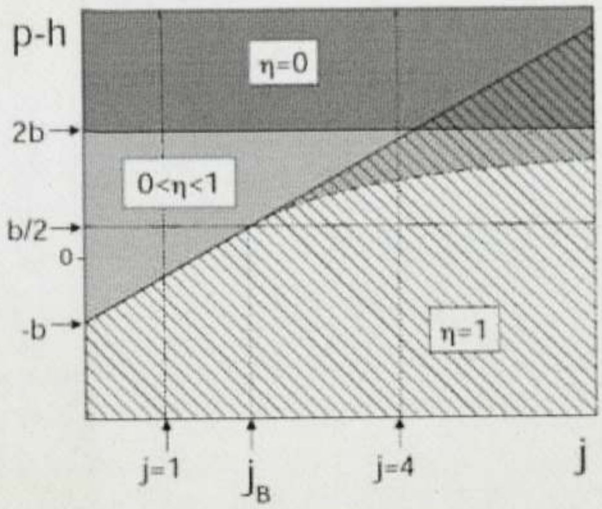


Figure 3.3: Customers Phase Diagram

If $j > j_B \equiv 3\sqrt{2}/2$, there is a range of prices for which two different solutions co-exist. One corresponds to a large fraction of buyers, and the other one to a fraction of buyers bounded by a finite upper value represented by the dashed line. Notice that for $p-h$ and j large enough, the two co-existing solutions are $\eta = 0$ and $\eta = 1$, due here again to the boundedness of the support of the IWP distribution.

3.3.3 Seller's Phase Diagram and The Seller's Dilemma

The monopolist want to find optimal solutions of the equation (2.27) in order to maximize his profit, i.e. to find out if he would rather sell to a large proportion of customer at a lower price or the only a small percentage of clients but at a high price. The result for p_{Max} depends only on the two parameters $h - c$ and j.

Following the work from V. Semeshenko and J.P. Nadal [22], we obtain that the possibility that two solutions coexist (as shown on the figure 3.4) put the seller into a dilemma : does he have to sell to more customers at a lower price, or to less buyers at a higher price? If the monopolist doesn't know precisely the parameters of the market (the distribution of H_i, the values of H, J, ...), he knows however that he will expect a phase diagram very similar to this one.

Practically, thanks to advertising for example, we may have an increase of H ("that's what you need") or of J ("everybody has it, so do not hesitate to get it"). However, because the first order phase transition is discontinuous, the seller can not guess when he should modify his price for a very important value to increase his profit. The question

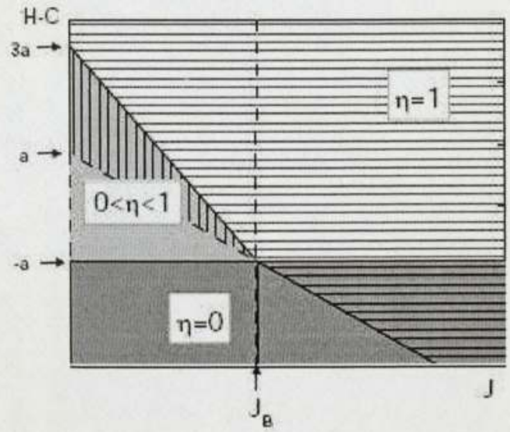


Figure 3.4: Seller's Phase Diagram

of whether the customers will actually buy or not is a coordination problem, whose issue depends on the dynamics of the adoption process.

The fraction of buyers, the optimal price and the corresponding monopolist's profit are functions of $h - c$.

Chapter 4

More Complicated Models

We examine in this chapter the case of more complicated models, for instance, the effect on the phase transition of introducing a small world structure in a regular lattice.

4.1 Small worlds

Following the paradigm of a "small-world" initiated by Milgram [15], Watts and Strogatz [26] proposed a formalisation in the field of disordered systems. Their original small-world starts from a regular network where n agents are on a circle (one-dimensional, periodic lattice) and each agent is linked with his $2k$ nearest neighbours.

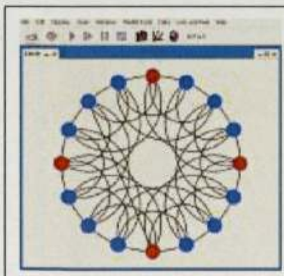
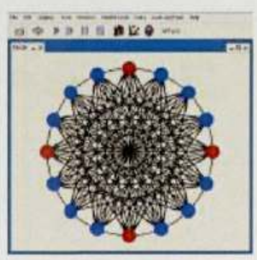
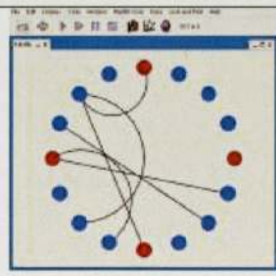
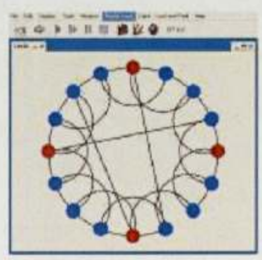
			
(a) Regular network connectivity k = 6	(b) Regular network full connectivity	(c) Random network random connectivity	(d) Small-world k = 4 and 3 links rewired

Table 4.1: Regular, random and "small-world" networks. (modeled with Moduleco)

In their rewiring algorithm, links can be broken and randomly rewired with a probability p. In this way, the mean connectivity remains constant, but the dispersion of the existing connectivity increases. For $p = 0$ we have a regular network and for $p = 1$ a random network. Intermediate values between 0 and 1 correspond to the mixed case, where a lower p corresponds to a more local neighbour-dependent network. The version of the algorithm implemented in Moduleco took h nodes, broke i links for each of these nodes and randomly rewired the broken links with other nodes.

Following Watt [26], two main structural indicators characterise a network through both the local and the global dimensions of its connectivity. These indicators use the language of graph theory: each node (agent) is called a "vertex" and each link an "edge". The connectivity of a vertex is the number of edges attached to the vertex.

In economics, the Small-World architecture has been applied by Jonard [12] to bilateral games, and markets models have been developed by Wilhite [28], among others.

For the spatial prisoner dilemma game, agents play a symmetric game with each of their neighbours on a lattice. In such a game, defection is the only *Nash equilibrium* of the one-shot game, and complex dynamics may arise, making the simulation very useful. In our game, at a given period of time, each agent plays the same strategy (S1 : co-operation or S2 : defection) and, at the end of the period, each agent observes the strategy of his neighbours and the average cumulated payoff. Jonard [12] has established for the *best average payoff* rule that the stability of cooperative coalitions depends on the degree of regularity in the structure of the network. In this example (Table 4.2), we have a co-operation in a regular network, one dimensional-periodic neighbour 4 structure (on a circle), but this cooperation is unsustainable : we make it sustainable by a rewiring disorder. We have a population of $N = 36$ agents (32 co-operators and 4 defectors), and the aim of this simulation is to improve the strength of a network against defection. The four *temporary* defectors are symmetrically introduced into the network. When the network is regular, defection is the winning strategy, and diffused quickly to the whole population.

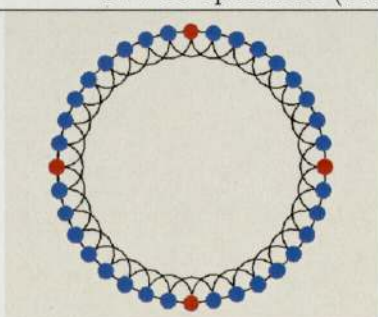
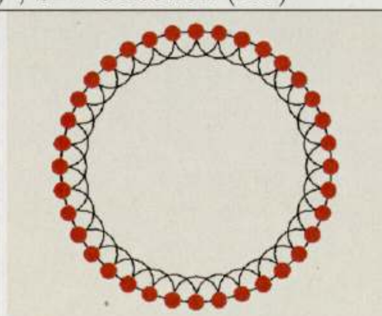
<table>
<tr><td colspan="2">S1 : co-operation (blue) ; S2 : defection (red)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>t= 0</td><td>equilibrium</td></tr>
<tr><td colspan="2">Within the regular network case, the number of defectors grew and became stable for 100% of the population</td></tr>
</table>

Table 4.2: Symmetric introduction of defection in a regular network of co-operators. (modeled with Moduleco)

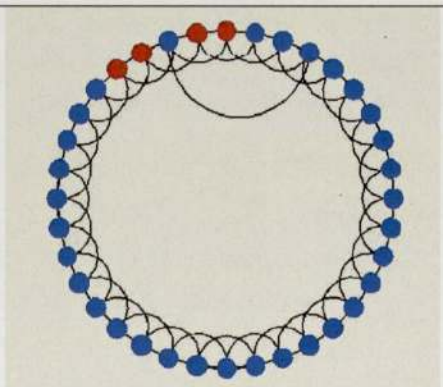
Without any rewiring, the system reaches an equilibrium with all the agents that do not want to buy any good.

Moreover, in roughly one half of the cases, defectors are limited to four or less. We ran 500 simulations and the first results (cf. Table 4.3) suggest that the percentage of stable co-operators becomes higher with sufficiently distant local neighbourhood. It means that the more distant the rewire is in the network, the less probability we have to end up with defectors.

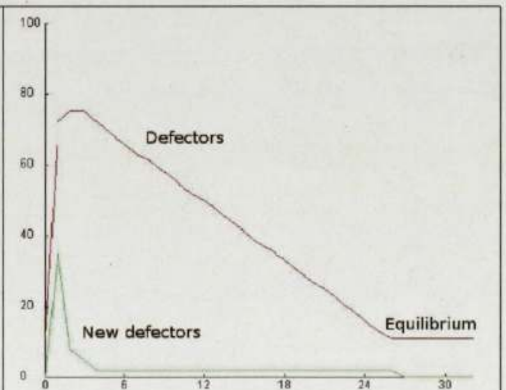
defectors	2	3	4	6	8	17	22	36
percentage	10.2	11.8	16.6	0.4	1.0	0.8	0.4	32

Table 4.3: Statistical results for 500 simulations.

In some cases, changes in the structure of the networks by minor modifications on the neighbourhood of some agents allow co-operation to protect against defection. In the previous example, taking the same symmetric system at $t = 0$, the number of defectors increases at first and reaches 70% of the population, but the rewired link, in some case, may reverse this evolution in a second step. In such a case (Table 4.4), defection decreases towards stabilisation around 10%. A one link rewiring is sufficient to limit to only 1/3 the percentage of cases with a totality or a majority of defectors.



equilibrium



evolution of the population

Table 4.4: Making the network more robust against defectors' invasion by rewiring only one link. (modeled with Moduleco)

But by only rewiring 2 agents, we can see on this figure on the left part that the equilibrium reached by the system is almost only composed of buyers and the evolution of the system (right part) with the dynamics of the system, i.e. the number of new defectors at each step (in green) and the total number of defectors at each step.

The only way to run simulations about small worlds in a realistic scenario (with empirical data) would be to have information about the location of the buyers or the sellers during the day or the date of the transactions. This information was not available in the data we gathered but it would be interesting to simulate this on a bigger market and see the impact of the geographical layout of the numerous agents.

The aim of this first part of the thesis was to analyse the theory and equations implemented in Moduleco, because the first aim of this project was to implement a tool to simulate and compare the behaviour of simulated agents and the real evolution of a market, the Marseille wholesale fishmarket in this case. That is the reason why the statistical analysis hereafter will not really have a direct link with the previous study, even if the Moduleco software was used to run almost all the simulations.

Chapter 5

A Multi-Agent Platform: MODULECO

5.1 Presentation of MODULECO/ACE

We used ACE (Agent-based Computational Economics) approach to investigate corresponding market mechanisms and underline in what way the knowledge of generic properties of complex adaptive system dynamics can enhance our perception of the market mechanism in the numerous cases where individual decisions are inter-related.

Moduleco [13] is a french modular "multi-agent" platform designed to simulate markets and organisations, social phenomenon and population dynamics. Moduleco was originally created by Denis Phan, ENST-Bretagne, France, and Antoine Beugnard, and is now maintained by Gilles Daniel, University of Manchester, UK, and Denis Phan. Moduleco is under GNU General Public Licence, so downloadable on the official website[1].

5.1.1 Why MODULECO ?

Moduleco is an object oriented modular framework, designed for the multi-agent simulations and using medium to formalise agent interactions.

The abstract sight of the world in Moduleco is adapted to a mathematical background and is a little bit different from the other multi-agent simulation platform (Ascape, Repast, Madkit).

The dynamics of the social system are based on the interactions that agents established together or with an environment external to the world. The interactions between agents are explained through the really important notion of "medium" that allows to build easily relations between agents.

The framework is hard to understand at the beginning because of its abstraction, but it guides a lot the models conceptor. It is implemented in Java, and runs on all platforms

[1]http://www.cs.manchester.ac.uk/ai/public/moduleco

with a Java Virtual Machine.

However, Moduleco provide not enough tools for the graphic interpretation of the results; the models conceptor has to code graphic outputs to handle it.

To conclude, the "economic and social sciences" orientation and the code's extension capacities make of Moduleco a perfect tool to realize a platform adapted to our needs.

5.1.2 *Reverse engineering* of the platform

The first step of the conception and development of a platform adapted to our study was the *reverse engineering* of the chosen platform, Moduleco. This step was crucial to understand its architecture and its way to work; it allows to explain the modifications and the improvements that we have to make in order to obtain the final platform.

A documentation about the existing Moduleco platform was made before looking into the code, but there wasn't a lot of papers about the architecture of the platform, this step was the longest because of the high complexity of the code.

To summarize, in Moduleco, a multi-agent system is made of a world represented by the class World. This world is an agent who belongs to the group "ecoAgent" in Moduleco and is considered as the "environment". This world is populated with an group of independent agents of which characterics and behaviour are defined in the class Agent. These agents live in the group "ecoAgent" in which they are considered as "basicAgent". Their disposition and the links between agents and their initialisation are made with a function of the class World. The communication between agents is made using the class Medium.

This class links two agents in an unique way giving the relation a particular role. An other type of agents can be defined: the "extra-agents". These agents are different from the "normal" agents and often appear as actors external from the system. This framework propose a set of classes allowing some abstraction and making easier the work of the models conceptor.

This UML Diagram summarize the logic of Moduleco's framework :

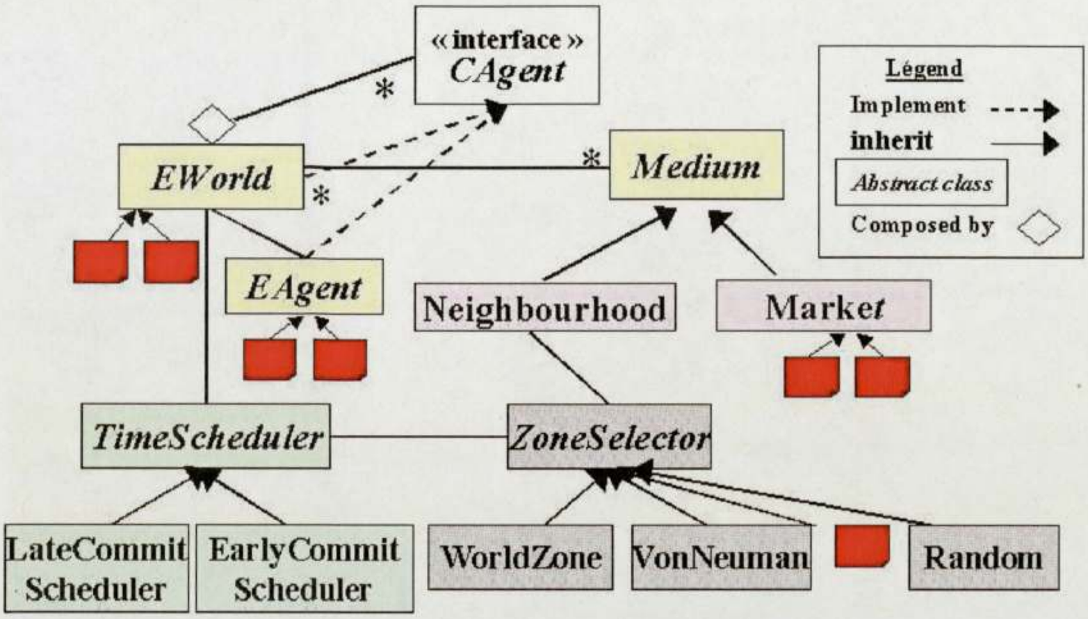


Figure 5.1: Moduleco's framework

This diagram summarizes the kernel classes of the Moduleco Framework. The yellow classes are the basic structural classes of Moduleco (Agent, World and Medium), the grey classes are related to the spatial relationship between agents (ZoneSelector, WorldZone, VonNeuman, Random), green boxes are related to the temporal aspects of the simulation (TimeScheduler, LateCommit and EarlyCommit) and pink boxes are concrete implementations of mediums and are related to the collaboration among agents, including spatial and temporal aspects. The red classes are the classes that the conceptor has to implement to create his model. By programing these classes, we can decide the particular behaviours for the world and the agents: predefined positions on the grid, neighbourhood, ...

5.2 Work on Moduleco

Meta-models are already built in Moduleco: they can be used as a guide in the development of new models built "from scratch" as solution to a problem or design need, or they can be more specified to correspond to the technology that the developer should use to be more specific to his problem. So Moduleco offers many models already developed that could be imitated, such as the *Two Part Tariff and Consumption Externality* model.

5.2.1 Modifications made on MODULECO

The basic-version of our model was strictly neo-classical monopoly with two part tariff and consumption externality, following Littlechild (1975). Consumers have variable willingness to pay, according to an idiosyncratic parameter, distributed on [0,1] and with an externality effect from their neighbourhood. In this simulations, we can change the neighbourhood type.

5.2.2 Simulations results

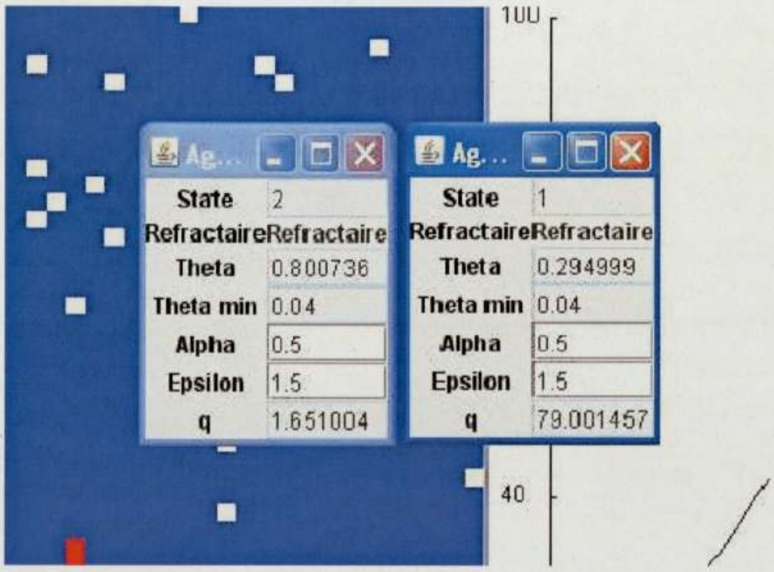


Figure 5.2: Agent Editor in *Two Part Tariff Competition*

The agent editor is a pop-up opening by right clicking on the agent. White zones are editable, grey one aren't.

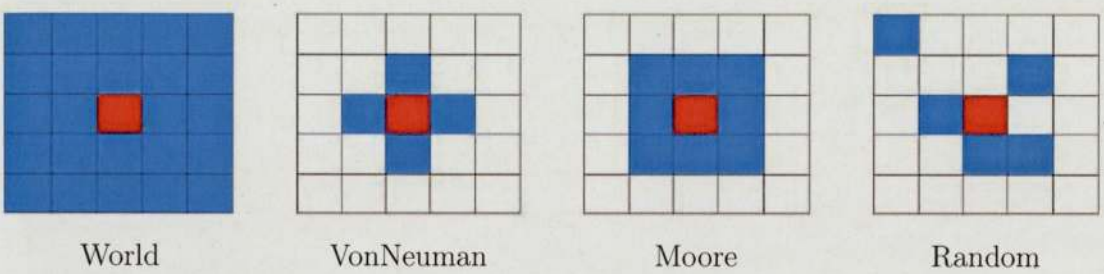
Recording the simulation results.

One of the fundamental functionalities of Moduleco is to make it possible to the user to observe results of simulations in graphic form. The conceptor can define in the model of the Java classes allowing to view the values of the variables. The graphic representations are charged in the interface during launching the model. The conceptor of the model can build himself these graphs, but he also can use the available functionalities. In this last case, only the data to be put up will have to be specified, which make the work of the conceptor much easier.

Furthermore, Moduleco allows us to record the outputs of the simulations, with the class "Recorder" which aims at recording the variables of the world and the agents specified by the final user thanks to the Graphic User Interface. These variables are then recorded in a cvs file (Comma Separated Value), interpretable by Excel. It is possible to do arithmetic operations on these variables, such as averages or sums.

5.2.3 Neighbourhood in Moduleco

In Moduleco, all relationships between agents are supported by specific Mediums. Such classes define how agents interact and how they are connected together. For example, NeighbourMedium allows Moduleco to define the set of neighbours an agent can have. Once his neighbourhood defined, an agent can invoke the services of his neighbours, such as getting specific information, for instance. Neighbours have specific subclasses for each specific topology such as WorldZone (all agents in the grid), NeighbourVonNeuman (North, South, East and West agents of the current agent on a grid) and Neighbour8 (the 8 closest agents on a circle). As a result, the communication topology is defined by the Neighbourhood. The grid is just an easy way to represent agents on a screen (that is offered by default, but that can be changed). For heuristic purposes, a circle representation is available, useful for the one-dimensional, periodic lattice.



A random neighbourhood is also available like with, for instance, a BoundedRandomZone topology. Finally, it is possible to change a regular network by rewiring some links, in the way of the "Small-Worlds".

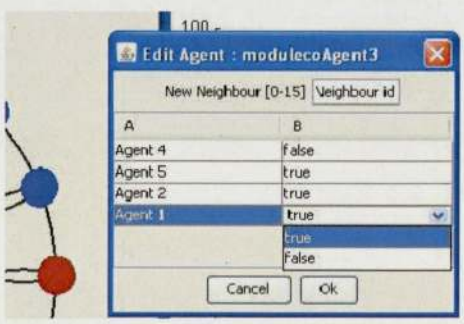


Figure 5.3: Neighbourhood Editor in *Moduleco*

We can edit an agent's neighbourhood by left clicking on the agent.

5.2.4 Avalanches using Moduleco

Results of numerical simulations on Moduleco permit us to illustrate the difference between localised avalanches and non-localised avalanches. In a system composed of 36 agents, the evolution of the number of customers is studied for different forms of neighbourhood. In the case where agents are isolated one from the others (no neighbours), the dynamic of the system is limited to 36 avalanches made up of only one agent. The social effect is null and the term "avalanche" does not seem to be really relevant for this case, it could be simply characterized as a spin flip. If agents are connected to two other agents ("neighbour 2"), the network is a circle, and, in numerical simulations, 13 avalanches were observed on average. For a "neighbour 4", the numerical simulations showed 9 avalanches on average. In these two cases, the localised effects of the avalanches are very clear because in each one, agents who modify their behaviour are in direct relation/connexion with the agent that precedes them.

In the other cases, that is, in the situation where all agents are connected one to the other ("world" neighbourhood), the agent composition of the 7 avalanches on average is dispersed on the network, and the local interdependence is replaced by a global interdependence.

The size of the largest avalanche is more significant in the last case where all the agents are connected one to the other.

Neighbours	0	2	4	world
Avalanches	36 spin flips	13	9	7

Table 5.1: Statistical results for 36 agents over 100 simulations.

Actually, the number of avalanches decreases with the size of the neighbourhood, while the size of the largest cascade increases. This result can seem obvious at first sight but it is not because it will totally depend on the configuration of the system and will change if we rewire some link in the network, for example.

This proposition is still true if we are changing the number of agents in the simulations:

Neighbours	0	2	4	world
Avalanches	45 spin flips	15	9	7
Avalanches	60 spin flips	23	10	7
Avalanches	80 spin flips	24	12	8
Avalanches	100 spin flips	28	13	8

Table 5.2: Statistical results for 45, 60, 80, and 100 agents over 100 simulations.

When we have an agent with an average willingness to pay, if, in his initial neighbourhood, he is surrounded by agents with a small willingness to pay, he is likely to purchase the good late (with a relatively small price). Increasing the number of neighbours decreases the risk of appearance of this kind of "frozen zone", the agent buys lately because he is not exposed enough to the social effect produced by his neighbours. On the contrary, if he is surrounded by agents who have a strong willingness to pay, he will buy the product rapidly.

The distribution of individual characteristics (willingness to pay) and the structural properties of the network of relations will influence the relative importance of the negative effect (frozen zone) and the positive effect.
In the simplest version of the simplest model, N agents play the symmetric game (prisoner dilemma) with each of their two neighbours on a circle (one dimensional periodic lattice). It exhibits a phase transition between two states : complete defection (nobody adopt the product) and almost complete co-operation (with a frozen zone).

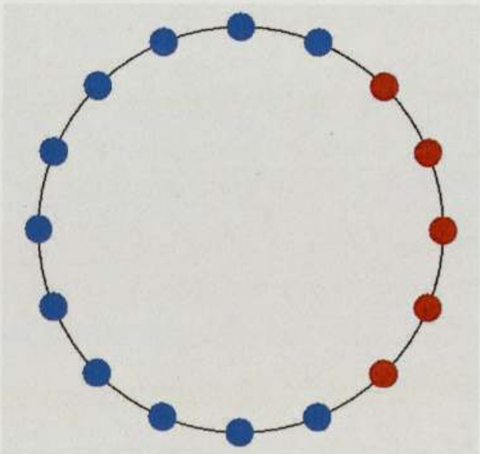


Figure 5.4: Example of a *"frozen zone"* of 5 agents for a one dimensional periodic lattice. (modeled with Moduleco)

5.2.5 Description of phase transition using Moduleco

The direct application of the theory studied previously in this thesis is to create a model to simulate the behaviour of a fixed number of agents (in this example we are using 1296 agents) and to observe the apparition of first order phase transition.

The development of this model was made by programming in Java and setting the different behaviour, but the graphic interface is hereafter very user-friendly and the other parameters can be entered manually when the application is opening.

This phase transition was illustrated shortly in the Chapter 3 and we are here explaining in detail the process of this transition.

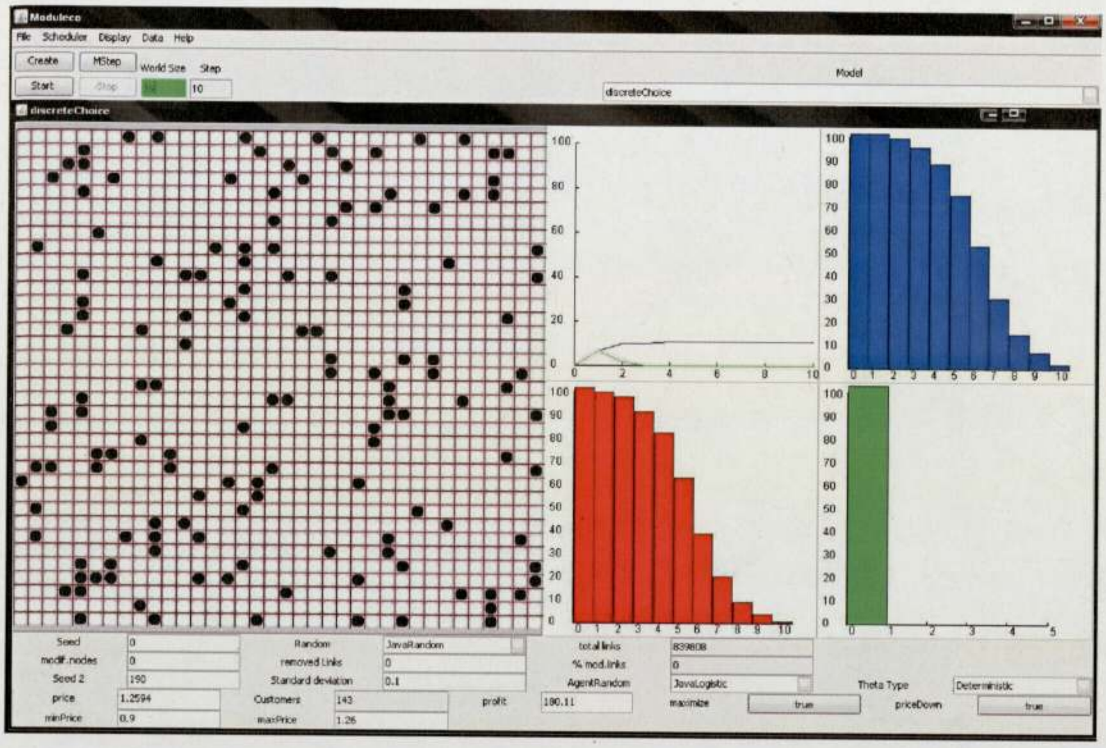


Figure 5.5: Phase transition: initial state. (modeled with Moduleco)

We randomly (JavaRandom) dispatch buyers in the population of the agents (figure 5.5). The price is initially 1.2594 and has to remain between 0.9 and 1.26, but it is evolving automatically depending on the willingness to pay of the agents. The evolution of the price is the simulation of the behaviour of the only seller of a monopolistic market. Rapidly, the number of buyers reach an equilibrium with approximately 10% of the agents deciding to buy the good.

The number of buyers is then increasing slowly and the price of the good is decreasing as well (see figure 5.6). We can observe below the graph of the penetration rate the cumulated logit distribution for idiosyncratic h_i, which is the willingness to pay without social influence. On the side of the graph of the penetration rate, there is the bar chart of the cumulated distribution for the total willingness to pay including social influence. These two graphs are very useful to make sure that the social influence is the major factor that will cause the phase transition.

But after 450 steps of the simulation, a large number of agents decide to buy almost at the same time. We are observing a first order phase transition. This transition is not directly due to the change in the price but to the social influence because the price changed constantly since the beginning of the simulation. This is so confirming the theory

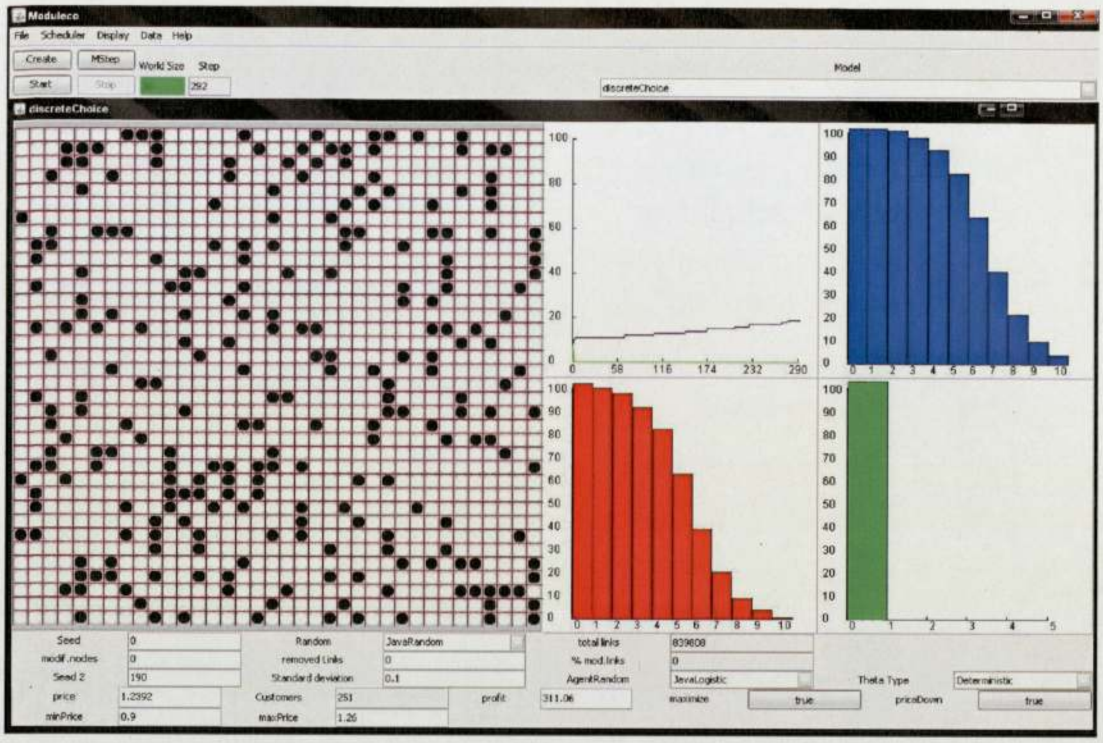


Figure 5.6: Phase transition: first equilibrium. (modeled with Moduleco)

analysed in the first part of this thesis. The green line on the graph corresponds to the the derivative of the penetration rate, or adoption rate, we can therefore observe a peak at the level of the phase transition.

The last figure represents the second equilibrium reached after this phase transition. This corresponds to a large amount of buyers (almost 90%) adopting the good at a lower price. This equilibrium is stable except if the price of the good starts to increase and then we would observe an hysteresis to come back to the initial state: a small number of buyers with a good at a high price.

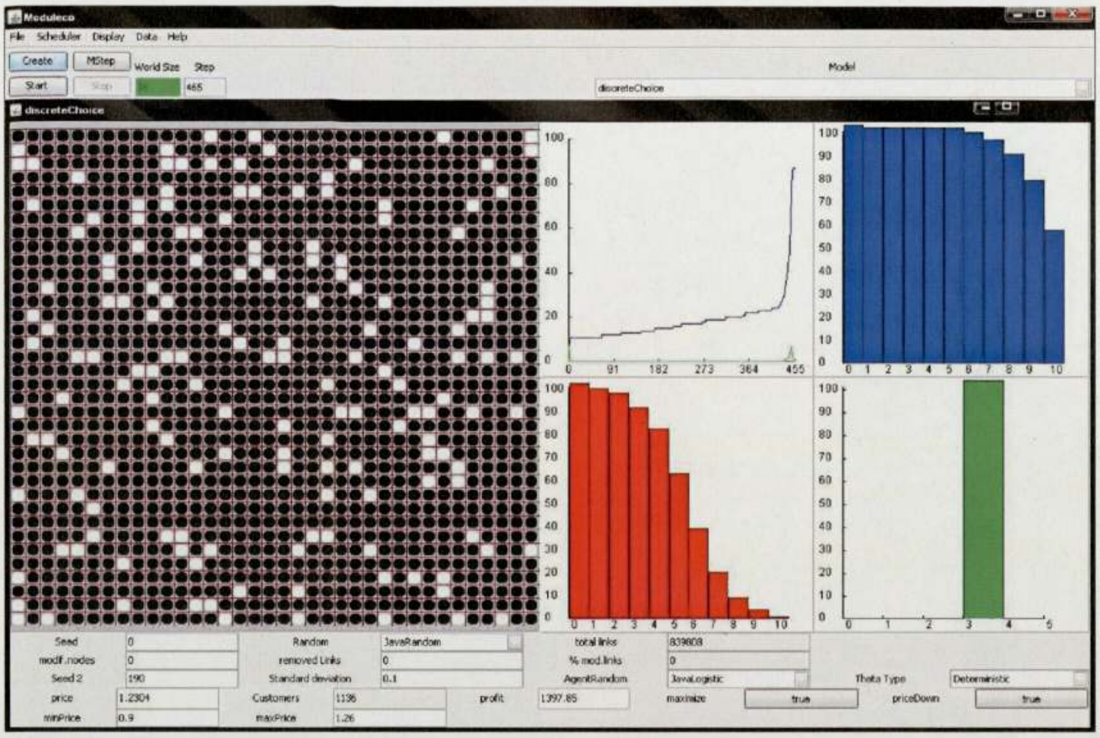


Figure 5.7: First Order Phase Transition. (modeled with Moduleco)

Figure 5.8: Phase transition: second equilibrium. (modeled with Moduleco)

Chapter 6

Empirical Evidence

6.1 Analysis of data from the Marseille fishmarket

In order to see whether there was any empirical evidence of ordered or disordered behaviour of buyers in a market, we started from a data base of the 237162 transactions that took place on the wholesale fish market in Marseille (M.I.N Saumaty). The particular interest of fish markets for economists is that they exhibit two features which make them a natural subject of analysis for economic analysis. Firstly, fish is a perishable good and the fact that, as a result, stocks cannot be carried over makes the formal analysis of the market simpler. Secondly the organization of such markets varies from location to location with little obvious reason (Marseille, France: pairwise trading; Sete, France: Dutch auction, i.e. descending price). On this market over 700 buyers meet over 40 sellers, to trade different types of fish. The market is organised as in our model, that is, no prices are posted, sellers start with a stock of fish which has to be disposed of rapidly because of its perishable nature. Buyers are either retailers or restaurant owners. Deals are made on a bilateral basis and the market closes at a fixed time. Of course the model is a caricature of the real situation since the alternative for a buyer to purchasing his optimal good is, in fact, to purchase , in his view, some inferior alternative.

The data base contains the following information :

700 buyers

40 sellers

And, for each individual transaction :

1. Name of buyer
2. Name of seller
3. Type of fish

4. Weight of fish bought

5. Price per kilo and total

6. Order in seller's transactions

This transactions took place from 02/01/1988 to 29/06/1991 inclusive.

Total number of transactions : 237162.

The sellers start with a stock of fish every day, and they have to sell it rapidly because of its perishable nature. Buyers are retailers or restaurant owners. The model is an extreme simplification of the real situation : there are different kinds of fish on the market, each species of fish is heterogeneous, buyers don't demand only one unit of fish and the alternative for a buyer to purchasing his optimal good is, in fact, to purchase some, in his view, inferior alternative.

In the following table, we can see an example with the number of transactions and the total weight for some transaction classes (or categories) where the column transac. relates the number of transactions in the period and kg. is the total weight of fish exchanged.

Transaction classes (TC)				Buyers			Sellers		
ID	Name	Transac.	kg	ID	transac.	kg	ID	transac.	kg
12	Small sole	3631	29449.99	160	3419	26755.51	52	1848	1282.25
13	Whiting	3084	23271.69	78	3281	25350.2	143	1781	12248.19
15	Small hake	3057	23575.2	1	2670	19177.11	44	1652	10764.82
10	Big sole	2563	16420.38	80	2584	19030.16	148	1651	11615.91
78	Mixed second choice	2332	11377.67	279	2027	14143.79	134	1623	10780.86
54	Big alive mantis shrimp	2258	14236.97	129	1795	13294.6	75	1515	11589.53
14	Medium hake	2248	17302.97	186	1539	11937.3	58	1411	9504.34
11	Medium sole	2065	15347.67	86	1190	8037.71	127	1404	8939.14
79	Big sea-hen	1849	12749.67	278	1179	9857.41	38	1361	10719.88
97	Cleaned mullet	1785	14958.5	75	957	3599.64	78	1356	10456.23
55	Small alive mantis shrimp	1647	9742.85	177	890	7129.71	59	1343	8658.34
56	Big stowed mantis shrimp	1539	13158.62	85	860	8416.43	118	1318	9043.4
100	Big angler fish	1375	7377.19	269	840	3762.47	43	1147	8774.42
70	Big scold fish	1262	6569.03	22	726	5878.52	100	1139	7199.81
2	Big cuttlefish	1200	9434.73	296	708	3788.93	54	1094	6482.03
58	Big pink shrimp	1190	4694.2	91	670	4447.75	138	1090	6741.73
17	Medium mullet	1168	11393.51	126	664	5008.23	83	1073	7559.3
16	Big Mullet	1121	13340.5	281	646	3638.49	64	1037	7542.84
42	Small polyp	1114	11401.07	237	625	4208.26	111	1030	7010.41
57	Small stowed mantis shrimp	870	6141.14	182	616	5263.99	95	1023	7096.78

Figure 6.1: Example of the database.

Using Microsoft Excel, we discover a lot of organisation in terms of prices and buyers preferences for sellers. In particular, we immediately observes that most frequent buyers, who visit the market more than once per week, visit only one seller, while less frequent buyers would visit several sellers, which is consistent with our model. Actually a frequent buyer tends to be more loyal to one shop than another, the frequency has to be considered as the organized regime. The transactions data will be summarized in this section in terms that only address the organisation issue.

Let examine the buyers and sellers size distributions. We define the size as the weight of the fish bought or sold by the agents. As shown on the figure below, there is no dominant size among the sellers while the buyers are clustered on the small size. The distribution of buyers presents a notable peak while that of sellers is rather flat. On the other hand it is obvious that there are a few very large buyers.

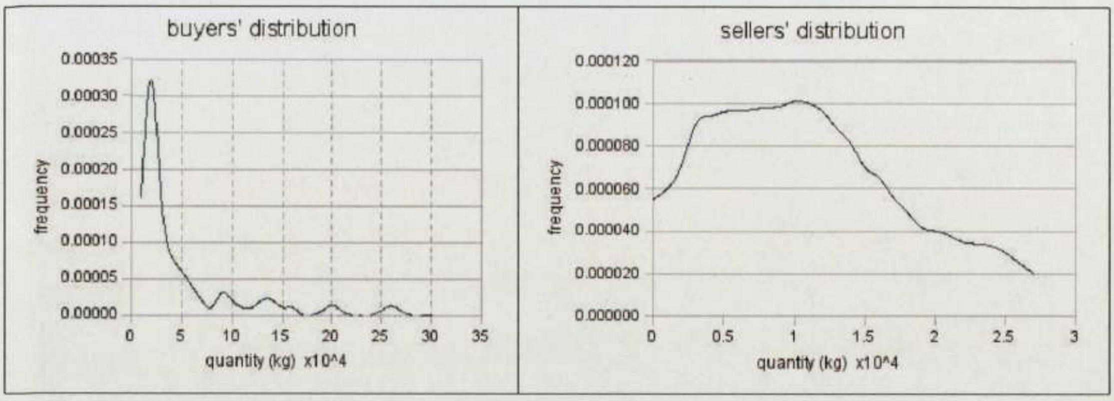


Table 6.1: Buyers and Sellers size distributions.

6.1.1 Loyalty in Marseille Fishmarket

At first, to compare the theory with the empirical data, we have to check whether individual buyers displayed ordered or disordered behaviour during the time of the recorded transactions. Since the classical approach to agent behaviour predicts searching for the best price, and since searching behaviour implies visiting different shops, any manifestation of order would tend to support our theoretical prediction. If we find evidence of ordered behaviour for certain buyers, a second step is then to relate the difference in the observed behaviours of these traders to some difference between their characteristics and those of other buyers.

Using the data and running them in excel, we obtain this

	market shares of largest sellers			monthly purchase share bought from one seller	
	1^{st}	2^{nd}	3^{rd}	$> 95\%$	$> 80\%$
cod	43%	14%	12%	48%	
trout	18%	7%	6%	28%	56%
sardine	20%	15%	15%	22%	52%

Table 6.2: Loyalty in Cod, Trout and Sardine Market

We consider statistics for cod, trout and sardine transactions in 1989, cf. Table 6.1.

We are interested in loyalty issues, so we have concentrated on the buyers who were present in the market for at least 9 months to allow order behaviour.
The first three columns of the table represent the percentage over the all market of cod, trout and sardine sold by the three more efficient sellers of the Marseille fish market whereas the last two columns represent the percentage of sellers who buy more than 95% and more than 80% respectively of their monthly purchases from one seller only.
As can be seen in the first three columns of the table, the market for cod is much more concentrated than the market for trout or sardine. In the cod market almost half the buyers (86 of 178) buy more than 95% of their monthly purchases from one seller only, as we can read in the fourth column of table. In the trout and sardine market, buyers are loyal too, but to a lesser degree : more than one half of them buy more than 80% from one seller. We observe that there are large fractions of loyal buyers in all three markets.

An other approach of this thesis was the connection between the behavior of the buyers and the parameters β (discrimination rate) and the cumulated profit π/γ. β surely vary from buyer to buyer, but we do not have any direct way given the data to test it *a priory*. We can however estimate the ratio π/γ by looking at the monthly purchases of the buyers.

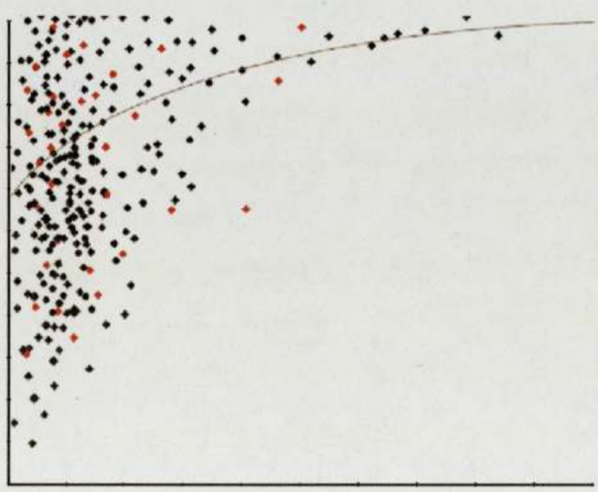


Figure 6.2: Each dot represents a buyer in Marseille fish market and its loyalty to his favorite seller (relative frequency of visits) as a function of his monthly purchase in cod. The horizontal axis represents the weight in kg of cod bought in a month and the vertical axis the frequency of visits.

Low purchases correspond to infrequent buyers whereas large purchase are those of buyers who visit nearly everyday the market.

6.1.2 Price dynamics

To analyse the price dynamics during the day we show two types of graph. We first rank the daily transactions by the time of day in which they occurred and then we perform averages for the transactions by the time of day in which they occurred and then we perform averages for the transaction with the same rank. As shown in the figure 6.3, the average price goes down as the rank of the transactions increases. A strange regularity appears: for a large number of transactions the average price starts increasing for the last transactions.

This apparent paradox can be understood linking the price stopping rule followed by the buyers to the relationship between last transactions variation in price and the quantity of fish of the day.

For many buyers, arriving at the transaction T, it may be optimal to buy even if the price is high when they have not reached the minimum quantity in order to satisfy customers demand. In days in which there is a low quantity of fish, it may be optimal to buy at higher than average prices starting from a number of transaction before the last one since with a limited supply it is likely that waiting for the last one will result in a difficulty to buy the needed fish.

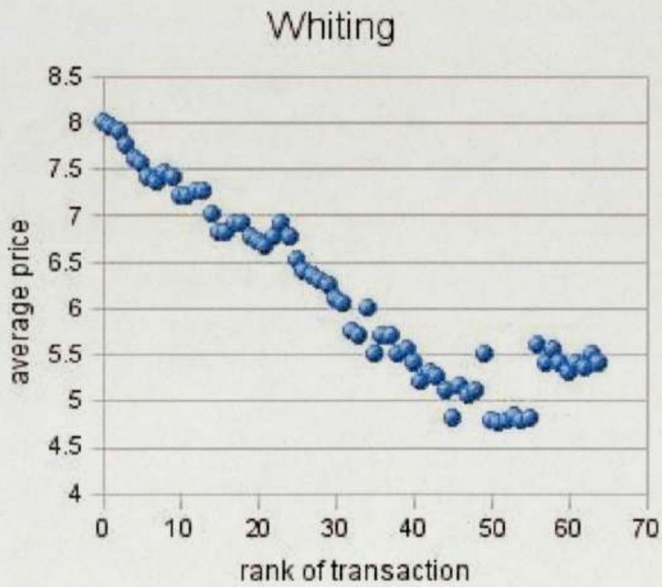


Figure 6.3: Average price for each rank of transaction.

We can link this observation to the theoretical work we reviewed in the first part of this thesis, and particularly the section about the mean field approach where we introduced an equation to obtain the average profit. We remind here this equation:

$$< \pi_j >= \pi P(j) \frac{exp(\beta J_j)}{\sum_k exp(\beta J_k)}, \tag{6.1}$$

This formula is the multiplication of the profit obtained from one actual transaction, the probability that the shop still has goods to sell when the buyer is coming and the probability that a buyer i visits a seller j. This equation may explain the fact of the high price at the beginning and at the end of the period of transactions: at the beginning of the period, all the sellers have the good needed (if we assume that there is only one type of good, otherwise it will depend on the type of fish), so the probability will be really close to 1. The only fact that incites the price not to go too up is the fact that there is a competition and so he has the choice between several sellers. The price afterwards reduces slowly because the sellers have a perishable good and so want to avoid to still have stock at the end f the period. On the other side, at the end of the period, the last term will be more influent because the buyer will not really have a lot of choices to find its special need and so the seller who still have good will be able to increase its price to sell the fish because he knows he can't really lose a lot of money as it could have been the case at the middle of the period.

6.1.3 Buyers and Sellers Price performance

The first question here is: are price performances related to the amount of fish transacted? The two figures below (6.4 and 6.5) will provide an answer. Basically it seems that the amount of fish bought or sold have no influence on price performances. This conclusion is robust for buyers while for sellers we can observe that large ones never sold at an average price lower than 7 euros, while some of the smaller sold at lower average price.

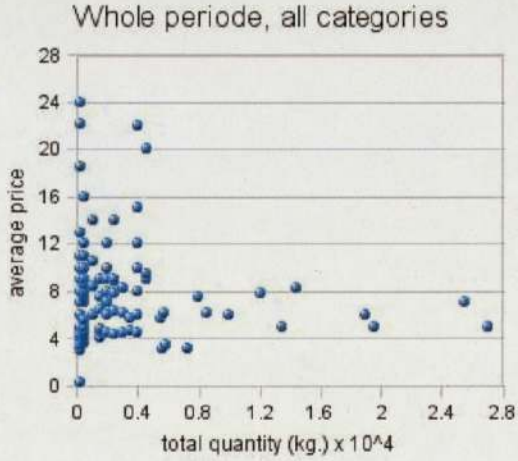


Figure 6.4: Buyers price performances.

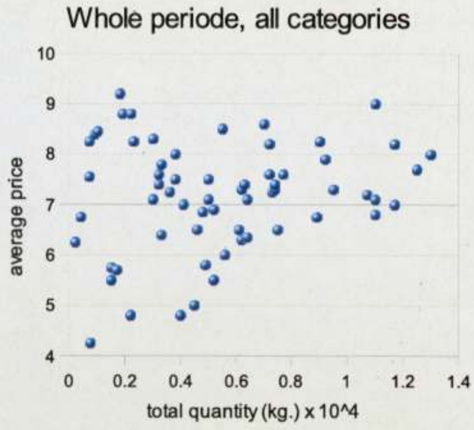


Figure 6.5: Sellers price performances.

So to find out if there are some buyers that pay higher (lower) prices than others we do a more sophisticated analysis. For each individual (buyer or seller), we calculate the monthly average price and rank the individual by its price, in increasing order. For each month, we associate the number one to the individual with the lowest price, two to the second, and so on... We are only doing this analysis over 9 months, so an individual that was always present on the market has a vector of 9 numbers attached denoting the ranks. If Mister X has the vector [12,5,...], this means that there were eleven other people with an average price lower than his average price on January, four people having a lower average price on February, ... To evaluate the performance of the subject we establish a threshold (for example 10) and count the number of times the rank of the individual was less or equal to the threshold. In the following table we denote with s the number of successes in this procedure, p the number of months he was present on the market. With r we identify the rank of the individual if we consider the average price on all his transactions over the whole period and all transactions, the price column is the average price and quantity is the total quantity. So from the following table we can infer that buyer 271 was present for all the 9 months; over the 9 months, his monthly average price was among the 10 lowest average prices. He bought 2991 Kg. of fish during the whole period at an average price slightly higher than 2 euros and this was the third lowest average price. But this result was not only on the overall market but true specifically for sole and whiting as well.

All Classes						Sole			Whiting		
id	s	p	r	price	quantity	id	s	p	id	s	p
164	9	9	7	3.077800	430	271	8	9	169	6	7
271	9	9	3	2.178703	2991.52	164	6	8	271	4	6
170	8	9	4	2.769652	1231.99	165	4	7	187	3	6
176	7	9	17	3.725731	2054.42	105	3	9	155	3	6
258	7	9	11	3.350421	581.84	221	3	7	237	3	6
153	7	8	10	3.309000	140.92	241	3	5	281	2	7
177	6	9	12	3.522101	7129.71	177	3	9	153	2	5
259	6	9	26	4.163261	317.77	101	3	7	138	2	7
169	6	9	16	3.678469	1449.74	178	3	6	1	2	7
1	5	8	13	3.579412	19177.11	278	2	5	130	2	7

Figure 6.6: Buyers with good performance.

Even if it could seem paradoxical, most of the buyers with the best performance are not always those that remain loyal to one seller but those who choose to follow the "mass effect". These agents seem more to respond to the influence of others around them than to their past experience as it could be observed by increasing the weight of J in simulations in Moduleco. They prefer to choose a seller that has already a lot of customers than to go to another one, even if he is cheaper. This could be explain by the fact that sometimes the quality of a fish could be indicated by the number of customers that are buying it in a particular shop. This behaviour seems to be adopted by several agents in the Marseille Fishmarket which could make think that these kind of agents are acting as a group and so be a sort of application of the mean-field theory. In the following table, we are showing the buyers with poor performance, which means that they are buying at very high prices. They all recorded a rank higher than 100.

All Classes						Sole			Whiting		
id	s	p	r	price	quantity	id	s	p	id	s	p
75	9	9	148	22.650290	3599.64	269	6	7	201	7	9
84	9	9	130	10.968370	1125.12	91	6	6	84	6	9
270	9	9	144	18.587760	1261.83	90	5	6	267	6	7
106	9	9	145	18.716570	427.93	263	5	6	90	6	8
188	8	9	141	17.103560	2235.15	199	4	7	279	5	9
195	8	9	137	13.973550	1818.57	273	4	6	186	5	9
82	7	9	139	16.181110	1634.21	162	4	7	91	5	7
148	7	9	132	12.090210	947.61	117	4	7	121	5	8
156	7	9	140	16.467580	788.25	217	4	7	87	5	8
281	7	8	138	14.803500	3638.49	267	4	7	272	4	6

Figure 6.7: Buyers with poor performance.

In the 2 next figures, we are focusing on sellers. a good performing seller wants to sell at high prices and so will have a high rank:

At the opposite, we have sellers with poor performance:

All Classes						Sole			Whiting		
id	s	p	r	price	quantity	id	s	p	id	s	p
138	6	9	66	8.802716	6741.73	148	3	8	30	6	7
83	4	9	48	7.745368	7559.3	81	3	5	123	6	6
148	4	9	60	8.315651	11615.91	139	3	6	66	5	9
121	4	8	58	8.230408	2130.33	83	3	8	67	5	9
30	4	7	61	8.349025	7239.03	134	3	7	145	4	7
126	4	9	55	8.056547	3779.51	100	3	5	22	4	7
146	3	7	65	8.499142	5170.71	38	2	7	138	4	7
45	3	7	51	7.910356	3315.49	122	1	7	118	4	8
134	3	9	69	9.051084	10780.86	113	1	7	37	4	9
66	3	9	63	8.394254	3173.71	129	1	7	105	4	7

Figure 6.8: Sellers with good performance.

All Classes						Sole			Whiting		
id	s	p	r	price	quantity	id	s	p	id	s	p
82	6	7	2	4.802500	2068.47	95	8	8	52	6	6
103	6	7	3	4.880414	3952.92	143	8	9	69	6	6
145	6	9	8	5.567217	5204.89	44	7	8	44	5	7
110	6	7	5	5.066325	4492.81	78	6	8	136	5	5
67	5	9	14	6.361449	6332.92	136	6	8	58	5	5
139	5	8	25	6.996613	5232.68	113	6	7	143	5	7
56	4	5	7	5.562541	872	48	5	5	83	4	5
113	4	9	10	5.832403	1489.01	58	5	8	129	4	8
130	3	9	11	5.837390	4837.3	134	3	7	148	3	5
48	3	5	1	4.142840	681.45	122	3	7	96	3	9

Figure 6.9: Sellers with poor performance.

6.1.4 Other models

The hypothesis of two main behaviors for the buyers seems to be confirmed by the loyalty section but it does not "prove" that this is the only possible model. As it is often the case with complex systems, several explanations at different level of generality can be used to describe observed phenomena. Furthermore, different models might not be mutually exclusive.

One other explanation can be that contractual arrangements are developing between buyers and sellers. In general in fish markets, the sellers do not offer fish for specific customers but "(the buyer) comes here because he knows that he will find the kind of fish he requires". Similarly, the buyers do not order fish, but they make the statement such as "I go there because he will have the fish that I want". This is consistent with the mutual reinforcement mechanism suggested by the theory (increasing of the IWP with the time in choosing one seller more than another). If a particular buyer does not appear, this is not regarded as a breach of contract and if this happens over a period and some quantity of fish remains unsold, the seller will simply readjust his supply of fish accordingly to avoid having a surplus at the end of the day.

We can find another explanation, based on the idea of "niches": a buyer would prefer a given seller because he provides him a product closer to his specific needs. Let us first note that the two hypothesis are not mutually exclusive: even if niches were an important factor, one would still have to explain why sellers choose a niche strategy rather than selling a large choice of fish. Loyalty of buyers might be a precondition for the profitability of "niches". Anyway, direct examination and surveys show that even though certain sellers specialise in serving supermarkets or institution cafeterias, almost all niches are occupied by several sellers. This is also consistent with the fact that many buyers are retailers who have to serve many different clients on their local markets. Another check for the existence of niches is clustering analysis according to average prices and quantities sold by sellers. Sellers are considered as members of the same cluster, when their distribution of prices and quantities significantly overlap. We did find two clusters of cod sellers, low cost bulk sellers (5 sellers) and expensive low quantity sellers (30 sellers). Since loyalty and search behaviour are observed in these two multi-member niches, the niche phenomenon cannot account by itself for the existence of loyalty; but according to our theory, it facilitates loyalty by decreasing the number of sellers in competition, and thus lowering the critical transition parameter.

The model we used, including its variants, considers buyers as active agents and sellers as rather passive. Alternative and/or complementary explanations of the observed organisation could be based on a more active role of sellers. A possible test of the necessity of extra hypothesis implying that loyalty is due to sellers' behaviour is to check whether different sellers have different fractions of loyal buyers among their customers, and if so, why.

By measuring the fractions of loyal buyers of each seller, it seems that they are strongly and positively correlated with the average quantity of fish per transaction sold by the seller (at least for all sellers making more than one transaction per day on average). The buyers learning and search behaviour as described in the model seems to be sufficient to explain the observed organisation and evolution, further assumptions about seller behaviour could be neglected for the model.

In the case of the wholesale fish market in Marseille, empirical data shows two kinds of buyers' behaviour : some buyers randomly choose the seller they will visit, and others have strong preferences, almost always visiting the same seller. By modelling it with a logit choice function, we can assume that each buyer has his own logit parameter β. As we have already seen in previous sections, phase transitions may occur as a function of β : buyers with β above the critical value β_c will most of the time select the same seller, and buyers with β below the critical value will continue to explore the most sellers as possible to find the right price, and this will occur even if all the profits are identical.

The logit choice function can be viewed as resulting from the maximization of a cost function which expresses a compromise between exploration (keeping information about the market) and exploitation (making the largest surplus at the next transaction). This can be understood either as the result of the search for an optimal mixed strategy by the agent.

Chapter 7

Conclusion

In this thesis, we have tried as far as possible to clarify the links with statistical mechanical models, with the aim of finding similarities between the model and the empirical evidence from the wholesale fish market of Marseille. We focused on a restricted class of models, because the literature of models of social influence in economics is ever growing and it is not possible to explore them all.

We have first examined a simple model of a market in order to see how the customers behave when the are influenced by social pressure to buy, or not, a single good in a monopolistic market. This kind of behaviour is observed on many markets, and especially for perishable goods.
Then, we have compared two special cases of the discrete choice model, the McFadden (McF) and the Thurstone (TP) models to show the differences between what the physicists call 'quenched' disorder models and 'annealed' disorder models.

In the simplest model, an ordered regime can appears, depending on the value of the agents discrimination rate, and when an individual's parameters put him into the organized regime, buyers have strong preferences for one shop over all others and the market is rather stable. On the other hand, in the disordered regime, agents do not show any preference and market performance exhibits large fluctuations. The transition between the ordered and disordered regimes is continuous but very abrupt (at least for the simplest model) in terms of number of buyers: t is what characterize a first order phase transition in statistical physics.

We then examined some papers already published about this subject and explored the theory about the link to statistical physics. Indeed, we tried to explain the Ising model in the field of physical properties of magnets. The equations were really similar to those developed in the first section of this thesis. The study of the different transitions that we can observe in physical processes as well as in agent's behaviour in a market allowed us to make a comparison between the different types of transitions: quenched and annealed disorder, avalanches and hysteresis effects. The phase transitions and the coexistence of several equilibrium states were then applied following the research done by V. Semeshenko [23] and using a triangular probability density function in order to create simple customers and seller phase diagram for the monopolistic market.

The introduction of more complicated models such as the Small-World initiated by Milgram [14] was an introduction to the influence of the connectivity of the network in its evolution. By only rewiring 2 agents, the evolution of the network could be totally different from the final equilibrium state without any rewiring.

We then presented the improvements developed on Moduleco, which was our principle tool to observe and analyse the different evolutions of the system given the initial parameters, the learning rules of the agents and the rules of progression of the components.

An analysis of selected data from the Marseille fishmarket shows the existence of a bimodal distribution of searchers and faithful buyers, depending on their idiosyncratic willingness to pay and so on the regularity they are purchasing a good and visiting a particular seller.

We have observed that in the simple model, and in others more realistic but without proper empirical data testing, in changing the connectivity of the network for example, that the presence of order, organisation and transition in a market is very dependent on the way in which agents react to their previous experience and to the reaction of their neighbourhood too.

This paper also explains the *declining price paradox* for the fish market of Marseille linking the price stopping rule followed by the buyers to the relationship between last transactions variation in price and the quantity of fish of the day. The average price tends to increase for last transactions in days characterised by limited (compared to customer's demand) supply of fish.

The first aim of the use of Moduleco was to try to create a proper tool in order to input directly the data gathered and run simulations to observe if the behaviour of the agents would be the same as it is in the real market. However, it was only possible for this project to run simulations on data added manually in the software. Some modifications are necessary to make the software totally efficient and so inputting data automatically which would make it a very useful tool to study this kind of interactions between agents and compare the outputs to real life markets.

To conclude, this thesis studied the similarities between statistical physics and agent-based macroeconomics and market dynamics. This approach of the financial markets is constantly in development and there are numerous papers and conferences appearing every month. We have learned different methods to analyse the behaviour of the agents in a social network but it seems that there is a constant wish to improve the simulation of these behaviours to make them more realistic and cloning the human behaviour. However, following our analysis of the Marseille wholesale fishmarket, it appears that there are still areas that deserve further research:-

- The development of Moduleco to allow the direct use of numerical data and so compare properly the results obtained with those of the real markets.
- The improvements of the models to obtain more human-like results in particular by avoiding using the simplest hypothesis and so characterizing a proper human behaviour in a market.
- The analysis of an other set of data, for example the establishment of a telecommunication network such as mobile phones or broadband membership. The advantage of the fishmarket is that data can be gathered every day and so the data base could be really wide.

Further work could be required in using other model than the mean-field approximation which is one of the simplest approximation, but this would complicate a lot the analysis of this kind of network and our aim is to simulate this model as real as possible but we still want to be able to analyse and interpret the results.

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References

[1] Begg D., Fisher S. and Dornbusch R. (2005) "Positive microeconomics", in *Economics - Eighth Edition.*

[2] Brock W.A. and Durlauf S.N. (2001) "Interactions-based models", in *Handbook of Econometrics*, edition 1, volume 5, chapter 54, pages 3297-3380.

[3] Camerer C.F. (2003) "Behavioral Game Theory", *Princeton University Press.*

[4] Durlauf S.N. (2001) "A Framework for the Study of Individual Behavior and Social Interactions", in *Sociological Methodology*, Volume 31, Number 1, pp. 47-87.

[5] Fudenberg D. and Levine D.K. (1998) "The Theory of Learning in Games", *MIT Press.*

[6] Galam S. (2004) "Spontaneous Symmetry Breaking", *in Bourguine (eds)*, pp. 157-168.

[7] Gordon M.B. (2004) "An Introduction to Statistical Mechanics", in Bourgine, Nadal (eds.), pp. 131-156.

[8] Gordon M.B, Nadal J.-P., Phan D. and Vannimenus J. (2005) "Seller's dilemma due to social interactions between customers", in *Physica A*, 356, Issues 2-4:628-640.

[9] Härdle W. and Kirman A. (1995) "Nonclassical demand, A model-free examination of price-quantity relations in the Marseille fish market", in *Journal of Econometrics.*

[10] Ising E. (1925) "Beitrag zur Theorie des Ferromagnetismus", in *Zeitschrift fur Physik*, 31, pp. 253-258.

[11] Jaynes E.T. (1957) "Probability Theory and Statistical Mechanics", *Phys. Rev. 106*, pp 620-630.

[12] Jonard N, Shenk E. and Ziegelmeyer A. (2000) "The dynamics of Imitation in Structured Populations", mimeo, BETA.

[13] Le Goff P. (2001) "Moduleco", *Lecture Notes for the IT students of ENST-Bretagne (France).*

[14] McFadden D. (1974) "Econometric Analysis of Qualitative Response Models", in *Handbook of Econometrics Vol II.*

[15] Milgram S. (1967) "The Small-World Problem", in *Psychology Today*, pp. 61-67.

[16] Nadal J.-P., Phan D., Gordon M.B and Vannimenus J. (2003) "Monopoly Market with Externality: an Analysis with Statistical Physics and ACE", paper presented at the 8^{th} Annual Workshop on Economics with Heterogeneous Interacting Agents.

[17] Nadal J.-P., Phan D., Gordon M.B and Vannimenus J. (2003) "Multiple equilibria in a monopoly market with heterogeneous agents and externalities", in *Quantitative Finance*, Volume 5, Issue 6, pp. 557-568.

[18] Nadal J.-P. and Gordon M.B. (2005) "Physique statistique de phénomènes collectifs en sciences économiques et sociales", in *Mathematical Social Sciences*, 43^{e} année, no 172.

[19] Phan D., Gordon M.B. and Nadal J.-P. (2003) "Social Interactions in Economic Theory: an Insight from Statistical Mechanics". in *Cognitive Economics: an Interdisciplinary approach*, Springer-Verlag.

[20] Phan D. and Pajot S. (2007) "Complex Behaviours in Binary choice Model with Global or Local social influence", in *Advance in Artificial Economics*, Springer Berlin Heidelberg, pp. 203-219.

[21] Selten R. (1983) "Evolutionary stability in extensive two-person games", in *Math. Soc. Sci. 5*, pp. 269-363.

[22] Semeshenko V., Gordon M., Nadal J.P. and Le Goff P. (2007) "Collective states in social systems with interacting learning agents", *as a part of the project ELICCIR.*

[23] Semeshenko V., Gordon M., Nadal J.P. and Phan D. (2007) "Discrete Choices under Social Influence: Generic Properties", *as a part of the project ELICCIR.*

[24] Smulders S. and Van de Klundert T. (2003) "Monopolistic Competition and Economic growth", in *The monopolitic Competition Revolution in Retrospect, Cambridge University Press.*

[25] Thurstone L.L. (1927) "Psychological Analysis", *American Journal of Psychology*, 38, pp 368-398.

[26] Watts D.J. and Strogatz S.H. (1998) "Collective Dynamics of Small-World Networks", in *Nature, Vol. 393/4, June.*

[27] Watts D.J. (1999) "Small Worlds, the Dynamics of Networks between Order and Randomness", in *Princeton Studies in Complexity*, Princeton University Press.

[28] Wilhite A (2001) "Bilateral Trade and Small-World Networks", in *Computational Economics*, volume 18, number 1, pp. 49-64.